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Mathematics 144, Winter 2017, Examination 1

Answer Key

1. [20 *points*] Suppose that we are given sets A, B, C such that $C \subset B \subset A$. Prove that $A - (B - C) \quad = \quad (A - B) \cup C$.

SOLUTION

. Suppose that $x \in A - (B - C)$. Then $x \in A$ but $x \notin B - C = B \cap A - C$. Since $C \subset A$, the second statement about x is equivalent to saying that x lies in the complement of $B - C = B \cap A - C$, which is $(A - B) \cup C$. Conversely, if x belongs to the latter set, the preceding reasoning implies that $x \in A$ but $x \notin B - C = B \cap A - C$.∎

2. [30 *points*] For each positive integer n let A_n denote the closed interval $[0, 1/n]$ and let B_n denote the open interval $(0, 1/n)$. Describe the sets

$$\bigcup_n A_n \ , \qquad \bigcup_n B_n \ , \qquad \bigcap_n A_n \ , \qquad \bigcap_n B_n$$

as intervals, one point sets, or the empty set.

SOLUTION

We have $\bigcup_n A_n = [0, 1]$, $\bigcup_n B_n = (0, 1)$, $\bigcap_n A_n = \{0\}$, $\bigcap_n B_n = \emptyset$.■

3. [*25 points*] Let $f : \mathbb{R} \to \mathbb{R}$ be the linear function $f(x) = 5x + 8$, and let A denote the closed interval $[0, 2]$. Find the image $f[A]$ and the inverse image $f^{-1}[A]$.

SOLUTION

The image $f[A]$ is the interval with endpoints $f(0) = 8$ and $f(2) = 18$.

The inverse image $f^{-1}[A]$ is the interval with endpoints a and b such that $f(a) = 0$ and $f(b) = 2$. Solving the equation $5x + 8 = C$ for $C = 0, 2$ we see that $a = -8/5$ and $b = -6/5$.■

4. [*25 points*] Let P be the set $\{a, b, c\}$ and suppose we are given some linear ordering on P (not necessarily the alphabetical ordering!). Prove that the ordering has a minimal element. [*Hint:* Since the ordering is linear, every two point subset has a minimal element.]

SOLUTION

Follow the hint. Consider the two point subset $\{a, b\}$. This has a minimal element, say x, such that $x \leq a, b$. Similarly, the set $\{x, c\}$ has a minimal element y for which $y \leq x, c$. It follows that $y \in \{a, b, c\}$ and $y \leq c, b, a$.■