Practice for Exam 1

Problems from aabUpdate04.144.f17.pdf

1. The unique possible choice A is a proper subset is $B = X$; it also works if $A = X$.

2. MISPRINT ALERT. The identity should be

$$\cancel{((X-A)\cap(X-(B\cup C)))^c = (A\cup(X-B))\cap(A\cup(X-C))}$$

~~Rewrite with $D^c = X\setminus D$~~

~~$LHS = (A^c \cap B^c \cap C^c)$~~

in the notation of the hint.

$$(A^c \cap (B\cup C))^c = (A\cup B^c)\cap(A\cup C^c)$$

Here is the verification.

L.H.Side $= A^{cc} \cup (B\cup C)^c = A\cup(B^c\cap C^c) =$

$(A\cup B^c)\cap(A\cup C^c)$, which is the R.H. side.

3. (i) ~~symmetric~~ reflexive since $|x-x| = 0$.

(ii) symmetric since $|y-x| < 1 \Rightarrow$

$|x-y| = |y-x| < 1$.

(iii) not transitive $|2/3 - 0| = 2/3$, $|4/3 - 2/3| = 2/3$

both <1, but $|4/3 - 0| = \frac{4}{3} > 1$.

4. $A \in \mathcal{P}(X)$ since ~~the latter~~ A is a subset of X.
We can also have $\{A\} \in \mathcal{P}(X)$ too; for example, if $X = \{\emptyset\}$. But not always, for $\{\emptyset\} \notin \mathcal{P}(\emptyset)$.

5. aRb true, $cRb \Rightarrow bRc$, so aRc
$aRc + cRd \Rightarrow aRd$
$aRd + eRd \Rightarrow aRd + dRe \Rightarrow aRe$
$aRe + eRf \Rightarrow aRf$
$aRf + gRf \Rightarrow aRf + fRg \Rightarrow aRg$.

6. Let $\mathcal{R}$ be the equivalence relation on $\{1,2,3,4,5,6,7,8,9,10\}$ whose equivalence classes are $\{1\}, \{2,3\}, \{4,5,6\}, \{7,8,9,10\}$.

7. We are given that R, S are equiv. relns.

<u>T is reflexive</u> xRx and $xSx \Rightarrow xTx$, all x.

<u>T is symmetric</u> Suppose xRy, xSy, give true. Then yRx & ySx are true (eq relns) $\Rightarrow$ so is yTx.

<u>T is transitive</u> Given $\overset{xTy}{\begin{matrix} xRy \\ xSy \end{matrix}}$ and $\overset{yTz}{\begin{matrix} yRz \\ ySz \end{matrix}}$.

The first choices $\Rightarrow xRz$, while the second $\Rightarrow xSz$, and hence xTz.

8. More generally, the sets A and $\{A\}$ are distinct, for if they weren't then we would have

$$A \in \{A\} = A.$$

<u>Stronger conclusion</u> $A \cap \{A\} = \emptyset$ always.