The relation between (*) and (**) typifies something that occurs quite frequently. All the remaining principles of set construction are pseudo-special cases of the axiom of specification in the sense in which (*) is a pseudo-special case of (**). They all assert the existence of a set specified by a certain condition; if it were known in advance that there exists a set containing all the specified elements, then the existence of a set containing just them would indeed follow as a special case of the axiom of specification.

If a is a set, we may form the unordered pair $\{a, a\}$. That unordered pair is denoted by

$$\{a\}$$

and is called the *singleton* of a; it is uniquely characterized by the statement that it has a as its only element. Thus, for instance, $\varnothing$ and $\{\varnothing\}$ are very different sets; the former has no elements, whereas the latter has the unique element $\varnothing$. To say that $a \in A$ is equivalent to saying that $\{a\} \subset A$.

The axiom of pairing ensures that every set is an element of some set and that any two sets are simultaneously elements of some one and the same set. (The corresponding questions for three and four and more sets will be answered later.) Another pertinent comment is that from the assumptions we have made so far we can infer the existence of very many sets indeed. For examples consider the sets $\varnothing$, $\{\varnothing\}$, $\{\{\varnothing\}\}$, $\{\{\{\varnothing\}\}\}$, etc.; consider the pairs, such as $\{\varnothing, \{\varnothing\}\}$, formed by any two of them; consider the pairs formed by any two such pairs, or else the mixed pairs formed by any singleton and any pair; and proceed so on ad infinitum.

EXERCISE. Are all the sets obtained in this way distinct from one another?

Before continuing our study of set theory, we pause for a moment to discuss a notational matter. It seems natural to denote the set B described in (*) by $\{x: S(x)\}$; in the special case that was there considered

$$\{x: x = a \text{ or } x = b\} = \{a, b\}.$$

We shall use this symbolism whenever it is convenient and permissible to do so. If, that is, $S(x)$ is a condition on x such that the x's that $S(x)$ specifies constitute a set, then we may denote that set by

$$\{x: S(x)\}.$$

In case A is a set and $S(x)$ is $(x \in A)$, then it is permissible to form $\{x: S(x)\}$; in fact

$$\{x: x \in A\} = A.$$