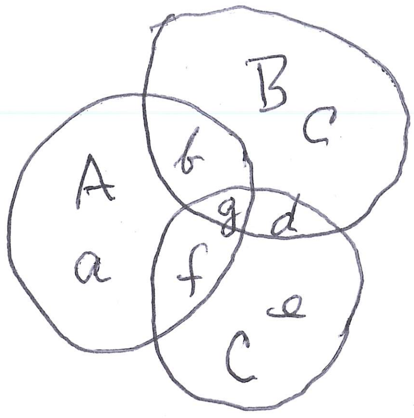
Quiz 1 Preparation

1. Here is a Venn diagram that might be helpful.



(a) First find $(A+B)+C$

$A+B = \{a, f, c, d\}$ $\Rightarrow$

$C = \cancel{\{b, c, d, g\}}$ $\{d, e, f, g\}$

$(A+B)+C = \cancel{\{a, b, f, g\}}$ $\{a, c, e, g\}$

Next find $A+(B+C)$

$A = \{a, b, f, g\}$ $\Rightarrow$

$B+C = \{b, c, e, f\}$

$A+(B+C) = \{a, c, e, g\}$.

(b) $(A+B)\cap C = \{d, f\}$

$A\cap B = \{b, g\}$

$A\cap C = \{d, g\}$

So

$(A\cap B)+(A\cap C) = \{d, f\}$.

2. (a) (a,x), (a,y), (b,x), (b,y) are the elements of $X\times Y$. There are 16 sets depending upon which elements contain them: $\emptyset$ [empty set]

$\{(b,y)\}$

$\{(b,x)\} \leftarrow \{(b,x)\}$

$\{(b,x), (b,y)\}$

$\{(a,y)\}$

$\{(a,y), (b,y)\}$

$\{(a,y), (b,x)\}$

$\{(a,y), (b,x), (b,y)\}$

$\{(a,x)\}$

$\{(a,x), (b,y)\}$

$\{(a,x), (b,x)\}$

$\{(a,x), (b,x), (b,y)\}$

$\{(a,x), (a,y)\}$

$\{(a,x), (a,y), (b,y)\}$

$\{(a,x), (a,y), (b,x)\}$

$\{(a,x), (a,y), (b,x), (b,y)\}$

[everything]

The system is to go from 0 to 15 using base 2 expansions. Each digit specifies if an element belongs to the set represented by the base 2 expansion.

(b) Finding $\mathcal{P}(A)$, $\mathcal{P}(B)$ and $\mathcal{P}(A\cup B)$ proceeds as before. Since $C\subseteq A$ or $B \Rightarrow C \subseteq A\cup B$, we have $\mathcal{P}(A), \mathcal{P}(B) \subseteq \mathcal{P}(A\cup B)$ and hence $\mathcal{P}(A)\cup\mathcal{P}(B)\subseteq\mathcal{P}(A\cup B)$. However, the containment is strict, for $A\cup B$ is not a subset of either A or B.

3. (a) $\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,6\}$
$\cancel{\{2,1\}}, \{2,3\}, \{2,4\}, \{2,5\}, \{2,6\}$
$\{3,4\}$ $\{3,5\}, \{3,6\}$ $\{4,5\}$ $\{4,6\}$ $\{5,6\}$

The key is to list the elements in order.

(b) Same idea

$\{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,2,6\},$
$\{1,3,4\}, \{1,3,5\}, \{1,3,6\}, \{1,4,5\},$
$\{1,4,6\}, \{1,5,6\}, \{2,3,4\}, \{2,3,5\},$
$\{2,3,6\}, \{2,4,5\}, \{2,4,6\}, \{2,5,6\}$
$\{3,4,5\}, \{3,4,6\}, \{3,5,6\}, \{4,5,6\}$.

4(First). $C \subseteq D$ is true,

for $6r-5 = 6(r-1)+1 = 3(2r-2)+1$

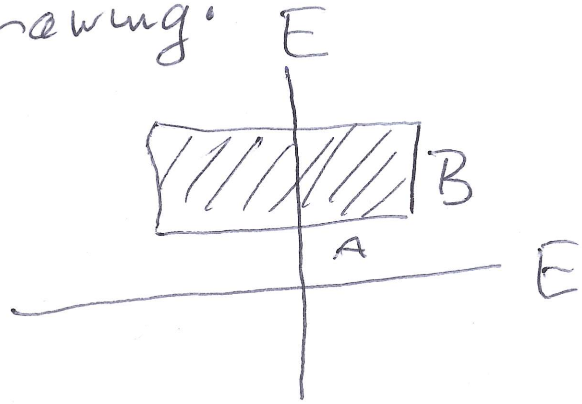
$D \subseteq C$ is false since $4 \in D$ but $4 \notin C$.

4(Second) If $x \in A \cap (B-C)$ then $x \in A$ and $x \in B$ but $x \notin C$. Hence $x \in A \cap B$ but $x \notin A \cap C$, so $x \in (A \cap B) - (A \cap C)$. Conversely, if $x \in A \cap B$ but $x \notin A \cap C$ then $x \in A$, $x \in B$ but $x \notin C$.

5. (a) $x \in A \cup C \Rightarrow x \in A$ or $x \in C \Rightarrow$ (since $A \subseteq B$) $x \in B$ or $x \in C \Rightarrow x \in B \cup C$.

(b) Same idea but substitute and & $\cap$ for or & $\cup$.

6. Drawing:



If $(x,y) \in E\times E - A\times B$ then either $x \notin A$ or $y \notin B$. Hence $E\times E - A\cup B \subseteq (E-A)\times E \cup E\times(E-B)$.
Conversely, if (x,y) belongs to either of the sets in the union, then either $x \notin A$ or $y \notin B$, so we also have the reverse inclusion.

7. The eight sets correspond to the 8 mutually exclusive possibilities:

Is $x \in A$? (Y/N)
Is $x \in B$? (Y/N)
Is $x \in C$? (Y/N).