USING THE

STRONG PRINCIPLE OF FINITE INDUCTION

Problem Show that every integer ≥ 44 can be written as $5p + 12q$, where p and q are nonnegative integers.

Inductive setup $P(n)$ is $1=1$ for $n \leq 43$.

If $n \geq 44$, $P(n)$ is the statement

$$\boxed{n = 5p + 12q \text{ where } p, q \text{ are nonnegative integers.}}$$

Then $P(n)$ is a tautology (hence true) if $n \leq 43$.

$P(44)$ is true because $44 = (4 \times 5) + (12 \times 2)$

$P(45)$ ——"—— $45 = (9 \times 5)$

$P(46)$ ——"—— $46 = (2 \times 5) + (12 \times 3)$

$P(47)$ ——"—— $47 = (7 \times 5) + (12 \times 1)$

$P(48)$ ——"—— $48 = (12 \times 4)$.

So $P(n)$ is true if $n \leq 48$.

Suppose now that $n \geq 49$ and $P(k)$ is true for all $k < n$.

Write $n = 5s + r$ where $1 \leq r \leq 5$, so that $n > n - 5 \geq 44$.

The latter inequalities imply that

$$n - 5 = 5p_0 + 12q_0 \quad \text{where } p_0, q_0$$

are nonnegative integers, so that

$$n = 5(p_0 + 1) + 12q_0, \quad \text{where } p_0 + 1 > 0, \; q_0 \geq 0.$$

Hence $P(n)$ is true.

This completes the inductive step in the Strong Principle of Finite Induction, and hence completes the proof.