# Mathematics 144, Winter 2022, Review for Examination 2

The examination will consist of five problems, most if not all of which are closely related to the ones given below.

**1.** Give an example of an infinite set $A$ and an infinite subset $C$ such that $|A| = |C| = |A - C|$.

**2.** Prove by induction that $2^n < n!$ for $n \geq 4$.

**3.** Prove by induction that

$$\prod_{k=1}^{n} \left(1 - \frac{1}{k^2}\right) \quad = \quad \frac{n+1}{2n} \ .$$

**4.** Let $A$ be the set of all continuous real valued functions on the unit interval $[0, 1]$, and define $f \ \mathcal{R} \ g$ if and only if the difference function $g(x) - f(x) \geq 0$ for all $x \in [0, 1]$. Prove that $\mathcal{R}$ is a partial ordering but not a linear ordering.

**5.** For each $k > 0$ let $P_k$ be the set of subsets $A \subset \mathbb{N}$ with exactly $k$ elements. Prov that $|P_k| = \mathbb{N}$.

**6.** Let $A$ and $B$ be finite sets with $p$ and $q$ elements respectively. Explain why the number of binary relations $\mathcal{R}$ from $A$ to $B$ (in other words $a \ \ b$ is true or false for each $a, b$) is equal to $2^{pq}$.

**7.** Prove that if $\alpha > 0$ is a cardinal number in a sufficiently large set $\mathcal{U}$ then there is a cardinal number $\beta$ such that $\beta^\beta > \alpha^\alpha$.

**8.** Let $S$ be a set and let $\mathcal{F} \subset \mathcal{P}(S)$ be a family of pairwise disjoint subsets such that each subset in the family has infinitely many elements. Prove that there is a subset $D \subset S$ such that for each $F \in \mathcal{F}$ the intersection $D \subset F$ has exactly two elements.

**9.** Recall that the disjoint union $A \sqcup B$ of two sets $A$ and $B$ is defined to be $A \times \{1\} \cup B \times \{2\}$. Prove that there is a 1–1 correspondence from $(A \sqcup B) \times C$ to $(A \times C) \sqcup (B \times C)$ where $A$, $B$, $C$ are arbitrary sets.