Solutions for Quiz 1

1. Let A and B be sets. Prove the absorption identity $A \cap (A \cup B) = A$.

SOLUTION

If $x \in A$, then x lies in both A and $A \cup B$, so $A \subset A \cap (A \cup B)$. Conversely, if $x \in A \cap (A \cup B)$, then $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ implies that either $x \in A$ or else $x \in A \cap B \subset A$. In either case we have $a \in A$, so that $A \cap (A \cup B) \subset A$. Since each of the two sets is contained in the other, the sets are equal.■

2. Let A and B be sets. Prove the absorption identity $A \cup (A \cap B) = A$.

SOLUTION

We know that $A \subset A \cup (A \cap B)$ because $A \subset A \cup C$ for any choice of C. Conversely, suppose that $x \in A \cup (A \cap B)$. Then either $x \in A$ or $x \in A \cap B \subset A$, so in either case $x \in A$. Therefore $A \cup (A \cap B) \subset A$. Since each of the two sets is contained in the other, the sets are equal.■