Math 144
Winter 2022

SOLVED PROBLEMS FOR WEEK 01

1. Give nonmathematical counterexamples to show that the following statements about set theoretic membership are not necessarily true for arbitrary sets A, B, C.

(i) $A \in A$.
(ii) If $A \in B$, then $B \in A$.
(iii) If $A \in$B and $B \in C$, then $A \in C$.

SOLUTIONS.

(i) We shall use the example of a deck of cards. Let A be the deck. Then the elements of A are single cards, and A is not a single card, so $A \notin A$.■

(ii) Suppose that A is a loaf of bread, so that the elements of A are slices of bread, and let B be a shipment containing loaves of bread, including A so that $A \in B$. Then $B \notin A$ because B is not a slice of bread.■

(iii) Let A be a slice of the loaf of bread B, and let B be one of the loaves in shipment C. Then $A \notin C$ because it is only a slice of bread and not an entire loaf.■

2. Give a nonmathematical example of sets A, B, C such that $A \subset B$ and $B \in C$ but $A \notin C$.

SOLUTION.

Once again let B be a loaf of bread in shipment C, and let A be some but not all of the slices of the loaf B. Only entire loaves are elements of C, so $A \notin C$.■

3. In the set theoretic approach to classical geometry, space is a set and the points are the elements of that set. Each line or plane will correspond to a subset of space. How might one interpret the concept of a line lying on a plane?

SOLUTION.

The appropriate interpretation of a line lying on a plane is that the subset given by the line is contained in the subset given by the plane.■

4. Suppose that **P**, **Q** and **R** are logical statements such that "Either **P** or **R** is true" is logically equivalent to "Either **Q** or **R** is true." Give a mathematical counterexample to show that this statement does not necessarily imply that **P** is logically equivalent to **Q**. Also, give a counterexample to show that the following analogous condition: "Both **P** and **R** are true" is logically equivalent to "Both **Q** and **R** are true." Give a mathematical counterexample to show that this statement does not necessarily imply that **P** is logically equivalent to **Q**.

SOLUTION.

Suppose that **P** is the statement that x is the real number zero, **Q** is the statement that x is the real number one, and **R** is the statement that x is a real number. Then both "**P** or **R**" and "**Q** or **R**" are logically equivalent to **R**, but certainly **P** is not logically equivalent to **Q**.∎

Similarly, suppose **P** is the statement that the integer x is a perfect square, **Q** is the statement that the integer x is a perfect cube, and **R** is the statement that the intege x is a sixth power. Then both "**P** and **R**" and "**Q** and **R**" are logically equivalent to **R**, but **P** and **Q** are not logically equivalent because there are integers that are perfect squares but not perfect cubes and vice versa.∎