Math 144
Winter 2022

EXERCISES FOR WEEK 02

0. Work the following problems from Cunningham:

Exercises 2.1 (pp. 36–37): 1–4, 16, 20, 24–26, 29–30
Exercises 2.2 (p. 40): 2, 4, 11

1. Let A and B be sets. Prove that the following are equivalent:

(i) $A \cap B = A$.
(ii) $A \subset B$.
(iii) $A \cup B = B$.

2. Let A, B, and C be subsets of a given set S. Prove that one has the mixed associativity (also known as **modularity**) property

$$(A \cap B) \cup C \quad = \quad A \cap (B \cup C)$$

if and only if $C \subset A$; in particular, the criterion has nothing to do with B. [*Hint:* This proof uses the distributive laws.]

3. Suppose that A, B, C and D are sets such that $A \subset C$ and $B \subset D$. Prove that $(A \cup B) \subset (C \cup D)$ and $(A \cap B) \subset (C \cap D)$.

4. Prove the following identities for Cartesian products:

(i) $(A \times B) \cap (C \times D) \quad = \quad (A \cap C) \times (B \cap D)$.
(ii) $(A \times B) \cup (C \times D) \quad \subset \quad (A \cup C) \times (B \cup D)$.
(iii) $(X \times Y) - (A \times B) \quad = \quad (X \times (Y - B)) \ \cup \ ((X - A) \times Y)$.

Also give an example in (ii) where the left hand side is not equal to the right hand side.

[*Hint:* It might help to draw rectangles corresponding to the various Cartesian products.]