Math 144
Winter 2022

EXERCISES FOR WEEK 06

The exercises in this document deal with material from Lecture 10 through Lecture 12.

0. Work the following problems from Cunningham:

Exercises 5.1 (pp. 115–117): 8 (with $\omega = \mathbb{N}$), 12, 16

Exercises 5.2 (pp. 122–124): 7, 14 [*Hints for* 14 : Show that the polynomials over $\mathbb{Z}$ of fixed degree n are countable, and show that a countable union of countable sets is countable.]

Exercises 5.3 (pp. 128–129): 1 – 4, 7 [*Note for* 7 : Recall that ${}^{A}B$ is the set of functions $\mathbf{F}(A, B)$.]

Exercises 5.4 (pp. 138–141): 1, 12, 16, 19, 34 [*Hint for* 1 : Find a function of the form $f(x) = mx + c$. — *Hint for* 12 : A previous exercise states that the function $f(x) = x/(1 + |x|)$ is a 1–1 onto map from $\mathbb{R}$ to $(-1, 1)$.]

1. Let n be a positive integer. Explain why the set A of all integers greater than or equal to $-n$ is well-ordered. [*Hint:* If B is a nonempty subset of A, consider the set C of all integers of the form $n + b$ where $b \in B$.]

2. Compute the number of ordered pairs (x, y) where x and y are integers between 1 and 10 such that one is even and the other is odd.

3. Prove that the set of countable subsets of the real numbers has the same cardinality as the real numbers themselves.

4. Suppose that X and Y are disjoint sets. Prove that there is a 1–1 correspondence from the disjoint sum $X \sqcup Y$ to $X \cup Y$.

5. Use the Axiom of Choice to prove the following statement: If $f : A \to B$ is a function, then there is a function $g : B \to A$ such that $f = f \circ g \circ f$.