

P.S. on the Strong Principle of Finite Induction

The condition $n \geq 44$ in `strong-induction.pdf` cannot be extended to $n \geq 43$. This can be seen by enumerating all expressions $5p + 12q$ where p and q are nonnegative integers such that $p \leq 8$ and $q \leq 3$; the constraints on p and q are needed for $5p + 12q$ to be less than 44.

Here is a list of possibilities. Note that none of the expressions below is equal to 43:

- $5p + 0 \cdot 12$ is a number ending in 5.
- $5p + 12 = 12, 17, 22, 27, 32, 37, 42$ and numbers ≥ 47 .
- $5p + 24 = 24, 29, 34, 39$ and numbers ≥ 44 .
- $5p + 36 = 36, 41$ and numbers ≥ 46 .

We can reorganize the preceding data to describe the positive integers not representable as $5p + 12q$ where p and q are nonnegative integers:

- No number ≤ 44 ending in 3 or 8 (9 non-representable examples).
- No number ≤ 35 ending in 1 or 6 (7 non-representable examples).
- No number ≤ 20 ending in 4 or 9 (4 non-representable examples).
- No number ≤ 10 ending in 2 or 7 (2 non-representable examples).

In contrast, standard theorems about the integers imply that every integer n can be written in the form $5p + 12q$ if we allow p and q to be positive, negative or zero. ■