Math 145A
Winter 2014

UPDATED GENERAL INFORMATION — JANUARY 28, 2014

The first midterm examination

The first midterm examination, which will take place on **Monday, February 3**, will cover everything through Chapter 5 in Sutherland as well as the portions of Chapter 6 dealing with limits of sequences and the basic definitions and properties of closed subsets (up to the last three lines on page 6.5 in the file `math145Anotes06.pdf`). The coverage of Chapter 6 corresponds to the subheading *Closed sets* and the portion of the subheading *Convergence in metric spaces* through the statement of Proposition 6.28.

The problems on the exam will be similar to the easy and moderately challenging exercises. Here are a few sample questions to consider. Some are probably more demanding than the problems which will appear on the exam but not dramatically so.

1. Let $f : X \to Y$ be a function of sets, and let B be a subset of Y. Prove that

$$B \quad \subset \quad f^{-1}\big[\,[f[B]\,]\big]$$

and give an example for which the containment is proper.

2. Let $f(x) = 1/x$ on the interval $(0, 2)$, and let $\varepsilon > 0$. Fine $\delta > 0$ so that $|x - 1| < \delta$ implies $|f(x) - f(1)| < \varepsilon$. It might help to analyze this as follows: If $\varepsilon > 0$ and $\varepsilon < \frac{1}{2}$, for what values of $x \in (0, 2)$ do we have

$$1 \;-\; \varepsilon \quad < \quad \frac{1}{x} \quad < \quad 1 \;+\; \varepsilon\,?$$

3. Let f be a monotonically increasing (but not necessarily strictly increasing) real valued function on the interval (a, b), and let $c \in (a, b)$. Define $f(c-)$ to be the least upper bound of all values $f(x)$ for $x < c$, and define $f(c+)$ to be the greatest lower bound of all values $f(x)$ for $x > c$. Prove that $f(c-) \leq f(c) \leq f(c+)$, and prove that f is continuous at c if and only if $f(c-) = f(c+)$.

4. In the real line give examples of subsets A, B satisfying the following conditions. [*Hint:* Try examples for which the subsets are intervals which may be open, closed or neither.]

(*i*) A is open, $A \cap B$ is open, but B is not open.

(*ii*) A is closed, $A \cap B$ is closed, but B is not closed.

(*iii*) Neither A nor B is closed, but $A \cap B$ is closed.

(*iv*) Neither A nor B is closed, but $A \cup B$ is closed.