Math 145A
Winter 2014

UPDATED GENERAL INFORMATION — JANUARY 26, 2014

The second midterm examination

The second midterm examination, which will take place on **Monday, March 3**, will cover everything from the material in Chapter 6 on closures up to (but not including) Proposition 12.19 in Sutherland (on the connectedness of closures of connected sets).

The problems on the exam will be similar to the easy and moderately challenging exercises, with more proofs and explanations than appeared on the first examination. Here are a few sample questions to consider. Some are probably more demanding than the problems which will appear on the exam but not dramatically so, and in some cases the problems in the examination may be simplified versions of some items below.

1. Suppose that X is a connected space which is the union of connected subspaces $A_1, \cdot, A_n$, where for each $i \geq 2$ we have $A_i \cap A_{i-1} \neq \emptyset$. Prove that $\cup_{i \leq n} A_i$ is connected. [*Hint:* For each k prove by induction that that $\cup_{i \leq k} A_i$ is connected.]

2. (i) A topological space X is said to have the Kolmogorov separation property if for each pair of distinct points in X there is an open subset containing one but not the other. Prove that if X and Y have the Kolmogorov separation property then so does their product $X \times Y$ (with the product topology).

(ii) Suppose that X and Y are two topological spaces in which each one point subsets is closed (the Frechet separation property). Prove that their product $X \times Y$ (with the product topology) also has this property.

3. (i) Suppose that X is a topological space and A is a subset which is not closed. Explain why A is a proper subset of its closure (in X).

(ii) Suppose that X is a connected topological space and U is a nonempty proper open subset of X. Prove that U is a proper subset of its closure (in X).

4. (i) Let $U \subset \mathbb{R}^2$ be the open first quadrant defined by $x, y > 0$. Find the set of limit points for U, and prove that your answer is correct.

(ii) Let $\mathcal{V}$ be the topology on $\mathbb{R}$ whose nonempty proper open subsets are the open rays $(c, +\infty)$ where $c \in \mathbb{R}$. Show that the rational numbers are dense in the topological space $(\mathbb{R}, \mathcal{V})$.

(iii) Explain why the topological space in (ii) is connected.

5. (i) A subset A of a topological space X is said to be locally closed if it is the intersection of a closed set and an open set. Prove that if A and B are locally closed in X, then so is their intersection $A \cap B$.

(ii) Suppose that X and Y are topological spaces such that $\mathcal{A} = \{U_\alpha\}$ and $\mathcal{B} = \{V_\beta\}$ are bases for the topologies on X and Y respectively. Prove that the set of all products $U_\alpha \times V_\beta$ is a base for the product topology on $X \times Y$.

(*iii*) Show that the topological space in $4(ii)$ satisfies the Kolmogorov separation property but does not satisfy the Frechet separation property.

6. Let $X = \mathbb{R}^2$, and let A be the set of points (x, y) in X such that either $y = 0$ or $x > 0$. Find the interior and boundary of A.