Math 145A
Winter 2014

UPDATED GENERAL INFORMATION — MARCH 14, 2014

The final examination

The final examination on Thursday, March 20, will consist of six questions, worth a total of 150 points and designed to be answerable in approximately 90 minutes (however, there will be three hours to complete the examination). Problems involving earlier material will be worth a total of 80 points, and problems involving Chapters 12 and 13 will be worth a total of 70 points.

As usual, the examination will involve knowledge of definitions, basic results, their meanings in simple cases, some logical derivations or proofs, and applications to specific questions which arise in single and multiple variable calculus. A very rough breakdown would be about 65 points for problems involving definitions *etc.* and what they mean for specific examples, 55 points for derivations and proofs (but note that one needs to know definitions and theorems in order to work such questions!), and 30 points for applying the basic concepts and results to specific questions about points and subsets in the real line or coordinate plane. To illustrate the latter, here are some (discarded) examples at the level to be tested:

> Let A be the subset of the coordinate plane consisting of all (x, y) such that either $x \geq 0$ or $y \geq 0$. Explain why A is connected.
>
> Describe all connected subsets A of the real line for which the interior of A is the open interval $(0, 1)$.
>
> Explain why the set of points (x, y) in the coordinate plane satisfying $|x| + |y| \leq 1$ is compact. [*Hint:* Why must we have $|x|, |y| \leq 1$, and why is the subset closed?]
>
> Explain why the set of points (x, y) in the coordinate plane satisfying $|x| \cdot |y| \leq 1$ is not compact.
>
> Let A be the set of all numbers $y = (3x + 4)/(5x + 6)$ where x ranges over all **positive** real numbers. What is the greatest lower bound of A? [*Hint:* Try graphing the function.]
>
> Let A be the set of all points (x, y) in the coordinate plane such that $y^2 = \pm 1$. Explain why A is not connected.

Regarding the material in Chapters 12 and 13 of Sutherland, the most important things to note are that connectedness provides an abstract setting in which one can prove an Intermediate Value Theorem for continuous real valued mappings, and compactness provides an abstract setting in which one can prove that a continuous real valued function has both a maximum and a minimum value. The characterizations for connected and compact subsets of the real line are fundamental, as is the characterization of compact subsets of coordinate n-space for all positive integers n. Criteria for recognizing compact and connected subsets are also extremely important (for example, unions of connected sets which have points in common, products of connected and compact spaces, and the relation between compact and closed subsets in a Hausdorff space).

Something involving the definition of continuity and verifying continuity for specific functions is likely to appear on the examination. One sample problem of this sort was included in the review for the first midterm examination. Another good thing to look at is the following: Suppose that $f(x) = mx + b$ where $m > 0$. Given $\varepsilon > 0$, find the corresponding $\delta > 0$; most of the time δ will depend upon both ε and the point x at which one wants to check continuity, but for a linear function (and other examples) it turns out that if ε is fixed then the same δ works for all choices of x.

Finally, the material covered on the examination does not go beyond Chapter 13 in Sutherland. In particular, there will not be anything about the topics discussed in `math145Anotes13a.pdf`.

Course and examination grades

After the final examination has been graded, solutions and the grading curve will be posted for those who are interested in seeing them. My general policy is that students are welcome to take and keep their final examinations, but for several weeks next quarter I will be unavailable. Students who want to retrieve their examinations should contact me by electronic mail so that arrangements can be made, possibly through the Department Office.