Addendum to `intro2topA-10.pdf`

Even though the unit disk U in $\mathbb{R}^2$ defined by $x^2+y^2 < 1$ is not a finite union of the rectangular sets $W(u,v)$, it IS a union of countably many such sets. At first this may seem to to contradict our geometric intuition, but the following argument shows that it is nevertheless true.

By Example 8.13 in Sutherland, we know that a countable base $\mathcal{B}$ for the topology of $\mathbb{R}^2$ is given by neighborhoods of the form $N_q(p)$ where q is rational and p has rational coordinates. If $x \in U$ let $W(x)$ be an open set of the form $W(u,v)$ which contains x, and let $N_{q(x)}(p(x)) \in \mathcal{B}$ be a basic open set such that $x \in N_{q(x)}(p(x)) \subset U$. Let $\mathcal{V} \subset \mathcal{B}$ be the family of all basic subsets obtained in this fashion, label them as $V_1, V_2,$ *etc.*, and for each i choose $W(x_i)$ such that $V_i \subset W(x_i)$. Then U is the union of the subsets V_i by construction, and therefore it is also the union of the subsets $W(x_i)$.■