Third Supplement to Chapter 12 of Sutherland,

Introduction to Metric and Topological Spaces (Second Edition)

If X is a finite set with n elements, then there are only finitely many possible topologies on X (a simple upper bound is 2 raised to the 2^n power), and in view of the material in this chapter one can ask how many of these topologies are connected. Similarly, in the setting of `intro2topA-07a.pdf` one can ask how many can ask how many *homeomorphism types* of topologies on X are connected. Here are the answers to these questions for the cases $n = 2, 3$:

If $n = 2$ then the discrete topology is the only one which is not connected.

If $n = 3$ then the only topologies which are not connected are the discrete topology, the three topologies in the homeomorphism type of the example whose nonempty proper subsets are $\{0\}$ and $\{1, 2\}$ (a union $A \cup B$, where A and B are disjoint open subspaces with the indiscrete topologies with $|A| = 1$ and $|B| = 2$), and the six topologies in the homeomorphism type of the example whose nonempty proper subsets are $\{0\}$, $\{1\}$, $\{0, 1\}$ and $\{1, 2\}$ (a union $A \cup B$, where A and B are disjoint open subspaces such that $|A| = 1$ and B is homeomorphic to the Sierpiński space).