

Addendum. Monotonic, Continuous functions of one real variable

The concepts and results of Chapter 12 allow us to prove the following basic theorem, which has been stated and used in examples and homework without proof.

monotonic
= either
increasing
or decreasing

THEOREM. Let $J \subseteq \mathbb{R}$ be an $\left\{ \begin{array}{l} \text{open} \\ \text{half open} \\ \text{closed} \end{array} \right\}$ interval,

and let $f: J \rightarrow \mathbb{R}$ be continuous and strictly (in/de)-creasing. Then $f[J] = K$ is an interval of the same type as J , and there is a continuous, strictly (in/de)-creasing function $g: K \rightarrow \mathbb{R}$ which is inverse to f . Specifically, $g[K] = J$ and we have $g(f(x)) = x$ ($x \in J$) $f(g(y)) = y$ ($y \in K$). ■

A proof of this result is given in the file [intro2topA-12a.pdf](#) in the course directory.