Summary of the Method

Goal Solve $f(x)=0$, where f is a continuous real valued function on some interval $[a,b]$ and we know that $f(c)=0$ for some $c \in [a,b]$. We want to evaluate c numerically in most cases.

Procedure Find another function $g(x)$ defined on some subinterval containing c such that

> Note: Finding g might be hard or not feasible. MAYBE IMPOSSIBLE

① $f(x)=0 \iff x=g(x)$ on the subinterval.

② $|g'(x)| \le \alpha < 1$ on the subinterval.

(these imply g maps the subinterval into itself).

Now form a sequence on the subinterval with x_0 chosen arbitrarily* and $x_{n+1}=g(x_n)$.

> *Often there are easy choices

Then the Mean Value Theorem and the Contraction Lemma imply that $c=\lim_{n\to\infty} x_n$.

Warning: If ② is false then $\lim_{n\to\infty} x_n$ might not exist, or possibly it exists but is not equal to c.