Math 145A
Winter 2020

UPDATED GENERAL INFORMATION — MARCH 11, 2020

Arrangements for the final examination

The examination will be a restricted take home test. You will have **three hours** to work it, but with **no books, notes, electronic devices, or input from others**. It will be posted on the course web site during the weekend, and it will be due **Wednesday, March 18, 2020,** at my office, Skye Hall mailbox, or UCR email account (`rschultz` *at* `ucr` *dot* `edu`) by **10:30 A.M.** If dropping this at the Department office (Skye 202), please remind the receptionist(s) that it should go into my mailbox. Regarding electronic copies, smart phone photos can be difficult or impossible to read, so any electronic submission should be easily printable and legible on a standard computer printer (*e.g.*, in `pdf` format as opposed to something that might not be available).

The examination will be posted in `pdf` format, and the easiest way to proceed is to download and print a copy. If you do not use the printed form, then you must include a cover sheet with your name and academic integrity certification (see below), and each problem should be started on a separate sheet of paper for the sake of efficient grading.

Academic integrity issues. You will be required to state the date on which you worked the exam together with starting and finishing times. It is implicitly assumed that submission of the exam is a confirmation that you have respected the rules in the first paragraph. Suspected violations can lead to procedures for handling academic dishonesty.

Missed third quizzes

Students who did not attend the afternoon discussion section due to confusion over the announcements by the Higher Administration will receive a quiz grade which is the average of the first two quizzes.

Practice problems and their solutions

Since the final exam is now a take home exam, full solutions to all the practice problems will not be posted. However, I shall post some hints to the 2020 problems together with a document on the older problems from earlier years.

The announcements by the Higher Administration have led to some changes in my previous draft of the final examination, and as part of the adjustment some additional practice problems are posted at the end of this document.

General comments

I apologize for the inconvenience that the modified procedures are likely to cause, but the whole situation is problematic so we have to do the best we can with available options. I also apologize if the requirements and integrity statements rub anyone the wrong way; this information needs to be sent out promptly, and unfortunately there isn't adequate time to sleep on everything and review the content for the sake of diplomacy.

Additional new problems

10. If (X, d) is a metric space, define the *closed* ε*-neighborhood centered at* x, written $CN_\varepsilon(x)$, to be the set of all $y \in X$ such that $d(x, y) \leq \varepsilon$.

(a) Verify that this set is a closed subset of X. [*Hint:* The function $g(y) = d(y, x)$ is a continuous function of y.]

(b) The previous part of the problem implies that the closure of $N_\varepsilon(x)$ satisfies $N_\varepsilon(x) \subset \overline{N_\varepsilon(x)} \subset CN_\varepsilon(x)$. Give examples of spaces X such that (i) the first two sets are unequal but the last two sets are equal, (ii) the last two sets are unequal the first two sets are equal, (iii) all three sets are unequal. [*Hint:* For each case, we can take X to be a subset of a closed interval in the real line with length 2ε such that X contains the midpoint and the two end points.]

11. Consider the following problem: *Let* $f : X \to Y$ *be a set-theoretic map of topological spaces, and suppose that* $\{A_\alpha\}$ *is a family of subspaces such that* $f|A_\alpha$ *is continuous for each* α*. Is* f *continuous?*

The answer turns out to be yes if each A_α is open in X or if each A_α is closed in X and the family is finite. Give an example to show that the answer can be no if $\{A_\alpha\}$ is an infinite family of closed subspaces. [*Hint:* Examples exist where the underlying spaces are equal, the topologies come from metrics, and $f(x) = x$ for all $x \in X = Y$. In other words, there are cases where one has metrics d, d' such that the condition in the problem is satisfied but $id_X : (X, d') \to (X, d)$ is not continuous.]