Math 145A
Winter 2019

UPDATED GENERAL INFORMATION — JANUARY 25, 2019

Office hours

These are currently scheduled for 10:30 to 11:30 on Mondays, and by appointment. The easiest way to make arrangements is to speak with me before or after class, or to send me an electronic message.

Assignments for Chapters $2-4$

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from Sutherland:

- Chapter 2: 2.1 – 2.4
- Chapter 3: 3.1, 3.3 – 3.6
- Chapter 4: 4.1 – 4.2, 4.8

The following references are to the file file `exercises01w14.pdf` in the course directory.

- Additional exercise for Chapter 3: 1
- Additional exercises for Chapter 4: 1 – 2

Reading assignments from solutions to exercises

Another strong recommendation is to read through the solution to Exercise 4.12 from Sutherland (see the file `solutions01w14.pdf` in the course directory).

Recommended exercises for Chapter 5 *of Sutherland*

- Chapter 5: 5.2 – 5.4, 5.6, 5.7, 5.9, 5.10, 5.13

The following references are to the file `exercises02w14.pdf` in the course directory.

- Additional exercises for Chapter 5: 1, 2, 5, $6(iv)-(v)$, 7

Quiz for Tuesday, January 29, 2019

The quiz will deal with the definition of metric spaces and determining whether a function $d : X \times X \to \mathbb{R}$ defines a metric on X. Here are some practice problems in addition to those in the exercises. The quiz problem will be less challenging and time consuming than some of those given below:

1. Let X be the set of all sequences $a = \{a_n\}$ taking values in $[0, 1]$, and define

$$d(a, b) \quad = \quad \sum_{n=1}^{\infty} \frac{|b_n - a_n|}{2^n} \; .$$

Show that the infinite series on the right hand side always converges and the formula defines a metric on X.

2. Let X be the set of all polynomials over the real numbers, and define

$$d(p, q) \quad = \quad \int_0^1 |p(t) - q(t)| \, dt \; .$$

Prove that this formula defines a metric on X. [*Hint:* If the right hand side is zero, why do we have $p(t) = q(t)$ for all $t \in [0, 1]$, and why does this imply that $p(t) = q(t)$ everywhere? Recall that polynomial functions are continuous.]

3. Suppose that (X, d) is a metric space such that $d(u, v) < \pi/4$ for all u and v. Prove that $\sin d(u, v)$ defines a metric on X. [*Hint:* Use trigonometric identities to show that $\sin(\alpha + \beta) \leq \sin \alpha + \sin \beta$ for $0 \leq \alpha, \beta \leq \pi/4$.]

Passive versus active understanding

In his essay "Of Studies," Francis Bacon (1561–1626) states that "Some books are to be tasted, others to be swallowed, and some few to be chewed and digested." In reading mathematical material, one way of interpreting this is that some things should be understood passively and others should be understood actively. Specifically, here is the difference between *passive understanding* and *active understanding*:

> A passive understanding means that one can follow the reasoning presented in a written proof fairly well.
>
> An active understanding means that one knows the argument well enough to explain it correctly — or nearly so — to someone else (for example, on a quiz or examination).

There are also many intermediate steps between purely passive and purely active understanding, but in this course we shall often note which things must be understood actively and which only need to be understood passively. However, in contrast to Bacon's statement, there are more than just a few things which need to be understood actively.