

**UPDATED GENERAL INFORMATION — FEBRUARY 14, 2019**

(Second posting on 2/14/19)

*The second quiz*

The second quiz on **Tuesday, February 19**, will cover the material corresponding to Chapters 7–10 of Sutherland. Some practice problems (in addition to the previously recommended exercises) are given below:

1. Let  $A \subset X$ , and assume that  $X$  has the indiscrete topology. If  $\mathcal{U}$  is the induced subspace topology on  $A$ , explain why  $\mathcal{U}$  is equal to the indiscrete topology on  $A$ .
2. Show that the open intervals  $(-\infty, b)$  and  $(a, +\infty)$ , where  $a, b \in \mathbb{R}$ , are a subbase for the standard metric topology on  $\mathbb{R}$ .
3. Let  $X$  be an infinite set with the cofinite topology, and let  $U$  be a nonempty open subset. Prove that the closure  $\overline{U}$  is equal to  $X$ .

In each of the following problems, determine if the statement is always true or sometimes true and sometimes false. In the second case, give examples where the statement is true and examples where it is false, using topological spaces with at least two points.

4. Let  $(X, \mathcal{U})$  be a topological space, and let  $\mathcal{U}'$  denote the sets of the form  $X - W$  where  $W \in \mathcal{U}$ . Then  $\mathcal{U}'$  defines a topology on  $X$ .
5. If  $\mathcal{U}_1$  and  $\mathcal{U}_2$  are topologies on a set  $X$ , then so is  $\mathcal{U}_1 \cup \mathcal{U}_2$ .
6. If  $X$  and  $Y$  are topological spaces with the cofinite topologies, then the product topology  $X \times Y$  is the same as the cofinite topology on this set.