NAME: ______________________________

Mathematics 145A, Winter 2020, Examination 2

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Return a hard copy to Skye 202 or 221 by 10:30 A.M. on Wednesday, March 18, 2020, with your start and finish times filled in as previously noted. No books, notes, electronic devices or other outside input are to be used when working this exam. You have up to three hours to complete this exam (different limits for individuals with certified disabilities). Thank you for your cooperation in working around the extraordinary and unanticipated circumstances

Starting date and time: ____________________

Finishing date and time: ____________________

#	SCORE
1	
2	
3	
4	
5	
6	
TOTAL	

1. [*25 points*] Let X be a topological space, and let A, B, C, D be connected subsets of X such that $A \cap B$, $B \cap C$ and $C \cap D$ are all nonempty. Prove that $A \cup B \cup C \cup D$ is connected.

2. [*25 points*] Let (X, d) be a metric space. Given $\varepsilon > 0$ and $x \in X$ define the closed neighborhood $CN_\varepsilon(x)$ to be the set of all $y \in X$ such that $d(x, y) \leq \varepsilon$. General considerations imply that this set contains the closure $\overline{N_\varepsilon(x)}$ of the open neighborhood $N_\varepsilon(x)$. Give examples of metric spaces where (a) these two sets are equal, (b) these two sets are unequal. [*Hint:* For the second one, there are simple standard subsets Y of the real line where $N_\varepsilon(x; Y)$ is a closed set.]

3. [*25 points*] If $\mathbb{Z}$ and $\mathbb{Q}$ are the integers and rational numbers respectively and $\mathbf{L}(\mathbb{Z})$, $\mathbf{L}(\mathbb{Q})$ denote their sets of limit points in the real numbers $\mathbb{R}$, then one of these sets is empty and the other is all of $\mathbb{R}$. State which one is empty and which is $\mathbb{R}$, and verify your assertion for either $\mathbf{L}(\mathbb{Z})$ or $\mathbf{L}(\mathbb{Q})$; you need not verify the other one.

4. [*25 points*] Suppose that X is a topological space with at least two points, and for every pair of distinct points $p \neq q$ in X there is a continuous a continuous function $f : X \to [0, 1]$ such that $f(p) = 0$ and $f(q) = 1$. Prove that X satisfies the Hausdorff Separation Property.

5. [*25 points*] Let X be a compact topological space. Prove X satisfies the following *Ascending Chain Condition* for open subsets:

> If we are given a sequence of open subsets $U_1 \subset U_2 \subset U_3 \subset \;\; ...$ such that $X = \cup_n U_n$, then there is some M such that $k \geq M$ implies $U_k = U_M = X$.

[*Hint:* Recall the proof that a compact metric space is bounded.]

6. [*25 points*] Suppose that (X, d) is a connected metric space with at least two points. Prove that there is a continuous real valued function f whose image contains a closed interval $[0, r]$ for some $r > 0$. [*Hint:* The metric d is a continuous function from $X \times X$ to $\mathbb{R}$.]

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