

EXTRANEOUS ROOTS

PROBLEM. Solve $x = 2 + \sqrt{18+x}$

Standard method $x - 2 = \sqrt{18+x}$) ★
 $x^2 - 4x + 4 = 18 + x$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

Hence $x = 7$ or -2 .

Check $7 \stackrel{??}{=} 2 + \sqrt{18+7} = 25$
 $= 2 + 5$. Yes, this is a solution.

$-2 \stackrel{??}{=} 2 + \sqrt{18-2} = 16 = 2 + 4$

No, this is not a solution.

Where is the problem? Usually in algebraic manipulations, passage from one line to the next is irreversible. However, the passage at ★ is irreversible. If $x^2 - 4x + 4 = (x-2)^2 = 18+x$, all we can say is that $x-2 = \pm \sqrt{18+x}$.