Homeomorphisms and properties of topological spaces

Homeomorphisms are specializations of a mathematical concept called isomorphism; more precisely, they are the version of isomorphism which is meaningful for topological spaces and continuous functions. The significance of the concept is that ***if two spaces are homeomorphic, then any well-formed statement about topological spaces is true for a given topological space is true if and only if it is true for a topological space which is homeomorphic to the given example.*** In other words, from a strictly topological viewpoint the spaces are (topologically) equivalent objects. A quotation which might be illuminating is given below. Specific examples of the italicized assertion appear in the review exercises for the third quiz.

> The interest in isomorphisms lies in the fact that two isomorphic objects have the same properties (excluding further information such as additional structure or names of objects). Thus isomorphic structures cannot be distinguished from the point of view of structure only, and may be identified. In mathematical jargon, one says that two objects are *the same* [*or equivalent*] *up to an isomorphism*.

https://en.wikipedia.org/wiki/Isomorphism

WARNING. Note that if two metric spaces are homeomorphic, then a well-formed statement about **metric spaces** might be true for one of the spaces but not the other. For example, the open unit interval and the real line are homeomorphic, but one of these metric spaces is bounded but the other is not. This reflects the fact that boundedness is not a well-formed concept for general topological spaces.