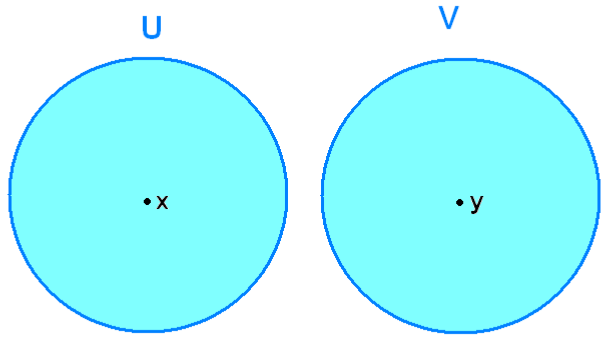
Supplement to Chapter 11 of Sutherland,

Introduction to Metric and Topological Spaces (Second Edition)

The Hausdorff Separation Property for Metric Spaces is apparent from the following drawing, in which **U** and **V** are open disks with centers **x** and **y.** The respective radii are **a** and **b,** where the latter are assumed to satisfy $a + b < d(x, y)$. This is slightly more general than the situation described in Sutherland.



For the sake of completeness, here is a proof that **U** and **V** are disjoint. If **z** is a point belonging to both open subsets then by hypothesis $d(z, x) < a$ and $d(z, y) < b$. Then the Triangle Inequality implies that

$$d(x, y) \leq d(x, z) + d(z, y) < a + b < d(x, y)$$

which is a contradiction. The source of this contradiction is the assumption that **U** and **V** have a common point, so no such point can exist.