strictly greater than

For discrete metrics,

$$N_r(x) = \{x\} \text{ if } r \le 1, \ X \text{ if } r > 1$$

For $\mathcal{C}([0,1])$ = continuous $\mathbb{R}$ valued functions,

$N_r(f)$ = all g such that $|g(x) - f(x)| < r$ all x.

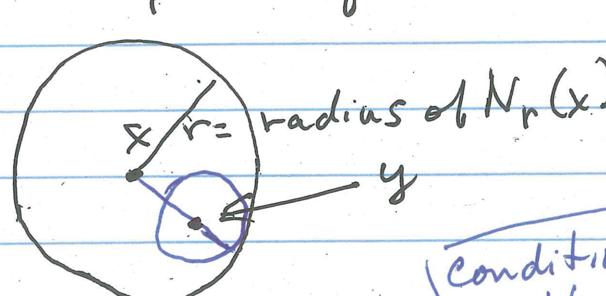
Open Sets

In $\mathbb{R}^n$, these are the sets needed to study partial derivatives.

__Def.__ (X, d^X) metric space. $U \subseteq X$ is __open__ in X (an open subset of X) $\Longleftrightarrow$ for each $u \in U$ there is some $\varepsilon > 0$ (depending on u) so that $N_\varepsilon(x) \subseteq U$.

__Sutherland, Prop. 5.31__ An open neighborhood of a point $x \in X$ is an open subset of X.

__Proof.__ Figure 5.4 on p. 53 of Sutherland is helpful.



It suggests we take $\varepsilon = r - d(x,y)$ if $y \in N_r(x)$.

condition for $z \in N_\varepsilon(y)$

We want $N_\varepsilon(y) \subseteq N_r(x)$. But $d(z,y) < \varepsilon \Rightarrow$

$$d(z,x) \le d(z,y) + d(y,x) < (r - d(x,y)) + d(x,y) = \varepsilon. \blacksquare$$

(so $z \in N_r(x)$)

Some open subsets of $\mathbb{R}$

(a,b) — If $x \in (a,b)$ then

$N_r(x) = (x-r, x+r)$, which is contained

in (a,b) if $r \leq x-a$, ~~b-y~~ $b-x$.

Similarly for $(-\infty, b)$ (need $r \leq$ ~~b-y~~ $b-x$)

(a,∞) (need $r \leq x-a$).

A subset which is not open in $\mathbb{R}$

$A = [0,1]$ is not open.

If $x = 0$, every set $N_r(x) = (-r, r)$

contains points not in A; specific examples

are $-\frac{r}{2}, -\frac{r}{3}, \ldots$.

More generally the $\left\{\begin{matrix}\text{closed} \\ \text{half open}\end{matrix}\right\}$ intervals

$\left\{\begin{matrix}[a,b] \\ [a,b) \\ (a,b]\end{matrix}\right\}$ are NEVER open in $\mathbb{R}$.