$$\int_a^b g(w(x))\,w'(x)\,dx = \int_{w(a)}^{w(b)} g(w)\,dw.$$

Linear algebra Geometric transformations of $\mathbb{R}^n$, $n = 2, 3, \ldots$

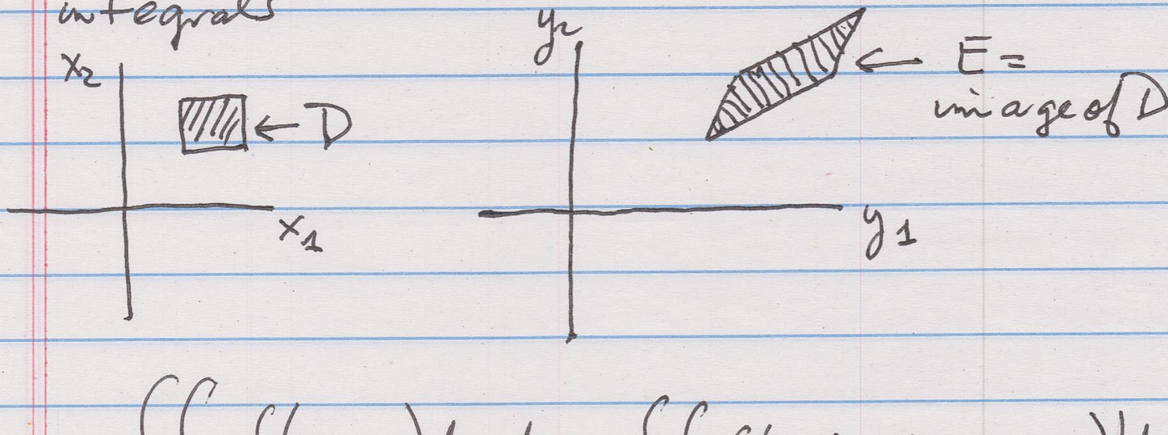
$$n=2 \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

where $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = A$ is invertible.

$Y = AX + B \implies$ inverse map is

$X = A^{-1}Y - A^{-1}B.$

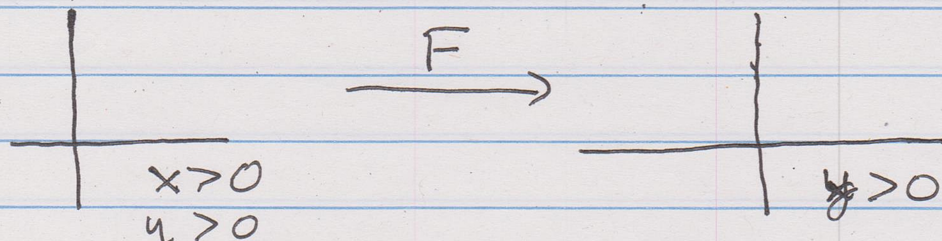
These can also be used to rewrite (double) integrals



$$\iint_E f(y_1, y_2)\,dy_1\,dy_2 = \iint_D f(y_1(x_1,x_2), y_2(x_1,x_2))\,|\det A|\,dx_1\,dx_2.$$

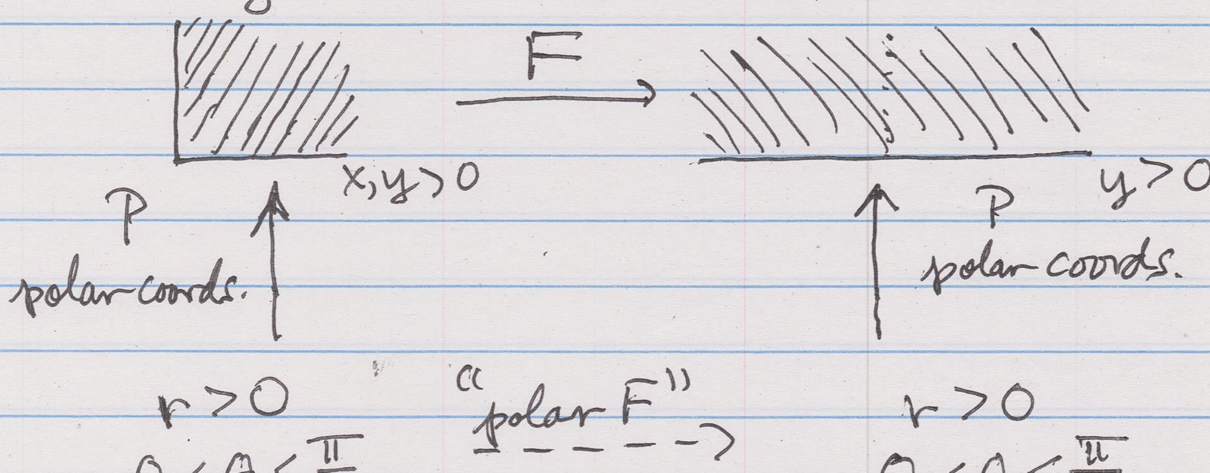
One also has more complicated 2D examples.

$$(u(x,y), v(x,y)) = F(x,y) = (x^2-y^2, 2xy)$$ on a piece of $\mathbb{R}^2$



In polar coordinates, F sends (r,θ) to $(r^2, 2\theta)$.

Checking that F has a continuous inverse.



The maps P have a continuous inverse

$Q(x,y) = (r,\theta)$ where $r = \sqrt{x^2+y^2}$

$$\theta = \arccos \frac{x}{\sqrt{x^2+y^2}}$$

In polar coords the inverse sends (r,θ) to $(\sqrt{r}, \theta/2)$.

We can write the inverse to F more explicitly as $P(\sqrt{r}, \frac{1}{2}\theta)$ where r and θ are given as above in terms of x and y.

See

<u>intro2topA-08.pdf</u>

for more on this topic.