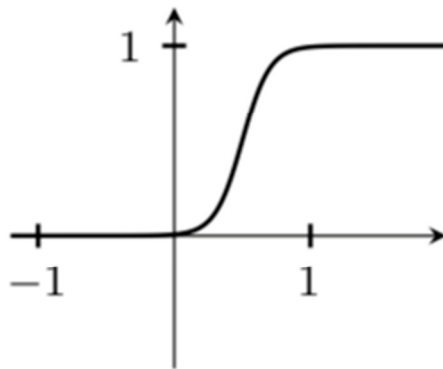


Illustrations for <http://math.ucr.edu/~res/math145A-2017/nicecurves.pdf>

Since pictures are very helpful for understanding the motivation and logic of the main result, we shall provide some here.

First of all, here is the graph of a so-called bump function which is 0 for nonpositive values of x , strictly increasing when x is between 0 and 1 , and 1 for all values of x which are at least 1 .

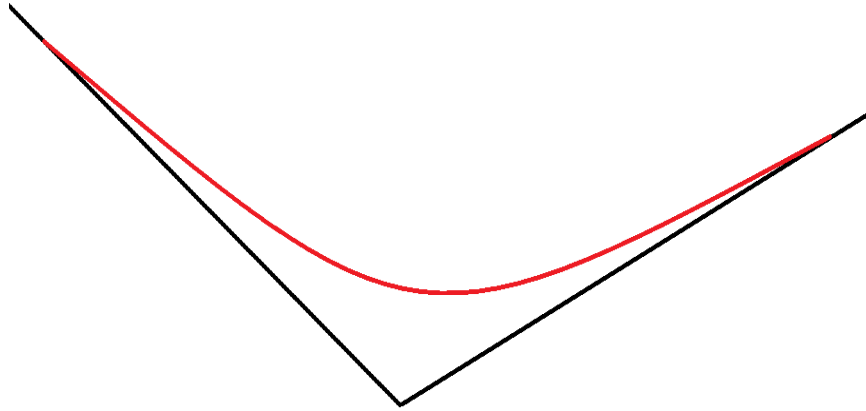


(Source: <https://i.stack.imgur.com/J0Ea3.png>)

At one point in the discussion there is an issue about concatenating (stringing together) two broken line curves such that the final segment of the first has the same direction as the initial segment of the second, and we note that such concatenated broken lines can be replaced by another broken line such that adjacent segments have linearly independent directions. The idea is to create a detour around the common point of the adjacent segments; the drawing below indicates how this can be done.



Finally, here is a picture suggesting that we can smooth out the broken line at corner points to obtain a new curve such that there is a well – defined and nonzero tangent vector on the (red) replacement piece:



There is an explicit description of how this can be done in the file cited above. The construction involves variants of the bump function whose graph is illustrated in the first picture of this file.