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Problems for Quiz 2

February 19, 2019

1. Let X be an infinite set, let $A \subset X$ be an infinite subset, let $\mathcal{U}$ be the cofinite topology on X, and let $\mathcal{W}$ be the induced subspace topology on A. Explain why $\mathcal{W}$ is equal to the cofinite topology on A. [*Hint:* If $B \subset A$ then $A - B = A \cap (X - B)$.]

2. Let $X = \{1, 2, 3\}$, and let $\mathcal{U}$ be the topology on X whose open sets are X, the empty set, and $\{1\}$. Give an example of a second topology $\mathcal{V}$ on X such that $\mathcal{U} \cup \mathcal{V}$ is not a topology for X, and give a reason why it is not a topology.