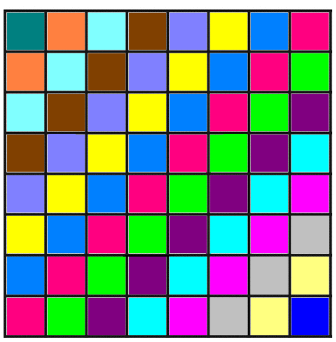
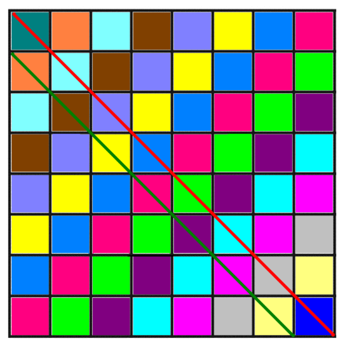
FIGURES FOR SOLUTIONS TO SELECTED EXERCISES

2. Notations and terminology

Additional exercises

1. Assume we label the squares by two positive integers on the board from left to right and from the bottom to the top. The first step in the argument is to show that a bishop can move diagonally to another square of the same color. In other words, if the bishop is located at the point whose horizontal coordinate is i and whose vertical coordinate is j, then the bishop can move one square up or down, to the square whose horizontal coordinate is $i+1$ and whose vertical coordinate is $j-1$, or to the square whose horizontal coordinate is $i-1$ and whose vertical coordinate is $j+1$, provided there are such squares on the board. Thus each diagonal lies in an equivalence class of points such that a bishop can move from one square to another in the class, and since there are exactly 15 diagonals in the drawing, this means there are at most 15 equivalence classes (see the drawing on the left).





The final step in the argument is to note that the points on the red and green lines in the right hand drawing also lie in the same equivalence class. Since the two lines contain exactly one square of each color, it follows that there are at most two equivalence classes of squares, and they are distinguished by whether $i+j$ is even or odd. In fact, there are exactly two such equivalence classes, for if the bishop moves one square from the position with coordinates i and j to a square with coordinates p and q, then by construction the sums $i+j$ and $p+q$ are both even or both odd.

See the file **http://math.ucr.edu/~res/math145A-2014/knight2014.pdf** for remarks on the squares that a knight on a chessboard can reach.