

How to use this website

There are four kinds of material in this website. First, we aim to make good the promises that certain proofs omitted from the book, and a few other topics, would be available on the website. Secondly, there are some sections which expand on topics in the book, or on concepts which are used in the book without any introduction. If you would like some help on such topics then this is for you. Thirdly, there is further material to enhance understanding and give more background. Finally, there are hints for starred exercises (and for a few others). Sections of the first three kinds are labelled to show which kind of material they contain:

- (1) = book promises
- (2) = explanations
- (3) = enhancement

The labelling may sometimes be a little subjective.

There is possibly more material in the website than is strictly necessary, but it is hoped that you will dip into it for reference and/or to read whatever catches your fancy.

The supplementary material associated with Chapter n is labelled $S.n$, and items within it are labelled $S.n.1$, $S.n.2$ and so on. Items from the book continue to be referred to by their number there. At the beginning of each $S.n$ there is a list of topics discussed in it. For convenience these are also listed at the end of this file. The page numbers given are those within each $S.n$. In addition there are two files of supplementary material not associated with chapters in the second edition of the book: $C.1$ is on compact subspaces of function spaces and $C.2$ is on completions.

It is suggested that you look through the list of contents of the website to see if anything you want to read more about is in there. In the last resort, if you cannot find what you want or it is not clear enough, please email suther@maths.ox.ac.uk and I shall try to do something about it.

The main point of this website (apart from the author's desire to drag himself into the 21st century) is to keep the book down to a length that a keen student might read almost all of.

As in the book, some sections are starred to indicate that they are thought to be more challenging. This does not usually apply to material labelled (2).

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