Math 145A
Winter 2016

UPDATED GENERAL INFORMATION — FEBRUARY 25, 2016

The third quiz

The quiz on **Tuesday, March 1,** will involve stating definitions and/or giving short answers for some of the following:

(1) State the definition of a/an (a) connected topological space, (b) arcwise connected topological space.

(2) State the characterization for connected subsets of $\mathbb{R}$ in terms of the standard ordering relation $a < b$.

(3) If A and B are connected subsets of a topological space X, give an example to show that $A \cup B$ is not necessarily connected, and state a simple but nontrivial condition for $A \cup B$ to be connected.

(4) State the Hausdorff Separation Property, which is the added condition in the definition of a Hausdorff topological space.

(5) Given an example of a topological space X and subsets $B \subset A$ such that B is not open in X but B is open in the subspace topology for A.

(6) Why does every nonempty topological space contain at least one connected nonempty subset? [*Hint:* We can find a FINITE subset with the required properties.]

(7) Given a positive integer n and a positive real number D, give an example of a connected subspace of $\mathbb{R}^n$ whose diameter is at least D. properties.]

(8) Given locally closed subsets L_1 and L_2 of X_1 and X_2 respectively, explain why $L_1 \times L_2$ is a locally closed subset of $X_1 \times X_2$. — The definition of locally closed subsets was stated in the (first) midterm examination.

(9) Using the standard spherical coordinates $0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq \pi$ and the conversion formulas $x = \cos\theta \sin\phi$, $y = \sin\theta \sin\phi$, $z = \cos\phi$, explain why the unit sphere in $\mathbb{R}^3$, which is defined by the equation $x^2 + y^2 + z^2 = 1$, is arcwise

(10) A *figure eight space* X is a union of two closed subspaces C_1 and C_2 such that each is homeomorphic to the unit circle and $C_1 \cap C_2$ consists of a single point. Explain why such a space is not homeomorphic to the unit circle. [*Hint:* Look at the complements of one point subsets.]