Math 145A
Winter 2016

UPDATED GENERAL INFORMATION — MARCH 4, 2016

(with minor corrections, Match 7)

The second in-class examination

This examination will take place on Friday, March 11. It will consist of four questions, and it will cover material beginning with `math145Anotes06b.pdf` and continuing through the end of the file `math145Anotes13c.pdf`. Half will involve the contents of Chapters 12 and 13 in Sutherland and the course notes, with the other half involving the contents of earlier chapters. Approximately 40 per cent will involve specific examples, and the remainder will involve more general considerations.

As usual, the examination will involve knowledge of definitions, basic results, their meanings in simple cases, some relatively simple logical derivations or proofs, and applications to specific questions which arise in single and multiple variable calculus. The assigned questions in the exercises, the contents of the second and third examinations from the Winter 2014 course, and the questions in the review for Quiz 3 (see `aabUpdate12.145A.w16.pdf` and `aabUpdate13.145A.w16.pdf`) are good sources of practice problems. Here are some further items for study:

(X0) Let A be the subset of the coordinate plane consisting of all (x, y) such that either $x \geq 0$ or $y \geq 0$. Explain why A is connected.

(X1) Describe all connected subsets A of the real line for which the interior of A is the open interval $(0, 1)$.

(X2) Explain why the set of points (x, y) in the coordinate plane satisfying $|x| + |y| \leq 1$ is compact. [*Hint:* Why must we have $|x|, |y| \leq 1$, and why is the subset closed?]

(X3) Explain why the set of points (x, y) in the coordinate plane satisfying $|x| \cdot |y| \leq 1$ is not compact.

(X4) Let A be the set of all numbers $y = (3x + 4)/(5x + 6)$ where x ranges over all **positive** real numbers. What is the greatest lower bound of A? [*Hint:* Try graphing the function.]

(X5) Let A be the set of all points (x, y) in the coordinate plane such that $y^2 = \pm 1$. Explain why A is not connected.

(X6) Let A and B be bounded subsets of a metric space X. Explain why $A \cup B$ is also bounded, and if $A \cap B$ is nonempty give an upper estimate for its diameter involving the diameters of A and B. Also, give examples where this inequality fails if $A \cap B$ is empty.

(X7) Let $f : X \to Y$ be a continuous function where X is compact and the topology on Y comes from some metric d. Explain why $f[X]$ is bounded, and if $y_0 \in Y$ explain why there is some point $x_0 \in X$ such that the distance function $d(f(x), y_0)$ takes a maximum value.

(X8) Suppose that (X, d) is a connected metric space and u, v are distinct points of X. Prove that there is some point w such that $d(u, w) = \frac{1}{2}\, d(u, v)$.

For Chapters 12 and 13, it is worthwhile to find examples where the criteria for recognizing connected and compact subspaces apply, and for Chapter 10 it is worthwhile to know examples where $B \subset A \subset X$ with the following properties:

> A is neither open nor closed in X, and B is open in A with respect to the subspace topology but B is not open in X.
>
> A is neither open nor closed in X, and B is closed in A with respect to the subspace topology but B is not closed in X.

For Chapters 8 and 9, it is worthwhile to know examples of topological spaces which do not come from metric spaces and to understand why the given examples cannot come from metric spaces (for example, one criterion for metrizability is that the one point subsets $\{x\}$ are closed for each $x \in X$).

As noted before, there will not be an examination during the week of March 14.

Course and examination grades

After the second examination has been graded, solutions and the grading curve will be posted for those who are interested in seeing them. My general policy is that students are welcome to take and keep their examinations. Students who want to retrieve their examinations should contact me by electronic mail next quarter so that arrangements can be made.