Math 145A
Winter 2017

UPDATED GENERAL INFORMATION — MARCH 3, 2016

The third quiz

The quiz on Tuesday, March 7, will involve stating definitions and/or giving short answers for some of the following:

(1) Give an example of a continuous 1–1 onto mapping of topological spaces $f : X \to Y$ which is not a homeomorphism.

(2) Give an example of a closed subset A of a topological space X such that $\overline{\mathrm{Int}\ A}$ (the closure of the interior of A, all in X) is a proper subset of A.

(3) State the Hausdorff Separation Property, which is the added condition in the definition of a Hausdorff topological space.

(4) If X is a Hausdorff topological space and $A \subset X$, why is the set of limit points $L(A)$ closed in X?

(5) Suppose that X is a topological space and A is a subset which is not closed. Explain why A is a proper subset of its closure (in X).

(6) What is the interior of the closure of the punctured interval $(-1, 1) - \{0\}$ (in $\mathbb{R}$)?

(7) Give examples of subsets $A, B \subset \mathbb{R}$ such that the intersection of the closures $\overline{A} \cap \overline{B}$ properly contains $\overline{A \cap B}$.

(8) Given locally closed subsets L_1 and L_2 of X_1 and X_2 respectively, explain why $L_1 \times L_2$ is a locally closed subset of $X_1 \times X_2$ if the latter has the product topology. — A subset $L \subset W$ is said to be locally closed in W if it is the intersection of an open subset in W and a closed subset in W .

(9) Suppose that X_1 and X_2 are topological spaces such that one point subsets in each are closed. Why are one point subsets closed in $X_1 \times X_2$ if the latter has the product topology? Conversely, if one point subsets are closed in $X_1 \times X_2$ if the latter has the product topology, why does the same property hold for X_1 and X_2?

(10) Given a Hausdorff topological space X and a point $p \in X$, let $\mathcal{N}_p$ be the family of all open subsets containing p. If p and q are different points of X, why are $\mathcal{N}_p$ and $\mathcal{N}_q$ distinct families of subsets?