Math 145B
Spring 2015

UPDATED GENERAL INFORMATION — APRIL 27, 2015

Still more about the first in-class examination

The following alternate characterization of Cauchy sequences will probably be useful:

CLAIM. *A sequence $\{x_n\}$ in a metric space X is a Cauchy sequence if and only if for each $\varepsilon > 0$ there is some positive integer N such that $n \geq N$ and $k \geq 0$ implies $d(x_n, x_{n+k}) < \varepsilon$.*

For the record, here a proof (for the exam it will suffice to know the characterization, and it will not be necessary to reproduce this proof): Suppose that the sequence is a Cauchy sequence, and let N be chosen so that $m, n \geq N$ implies $d(x_m, x_n) < \varepsilon$. Let p be the smaller of m and n, and write the other integer in the form $p + k$, where $k \geq 0$. Then $p \geq N$ and $k \geq 0$ imply $d(x_p, x_{p+k}) < \varepsilon$. Conversely, suppose that the condition in the Claim is satisfied, and choose N such that $p \geq N$ and $k \geq 0$ imply $d(x_p, x_{p+k}) < \varepsilon$. If $m, n \geq N$ then we know that $\{m, n\} = \{p, p + k\}$ for some $p \geq N$ (the smaller of m and n) and $k \geq 0$ (so that $p + k$ is the larger of m and n). Therefore $m, n \geq N$ imply $d(x_m, x_n) = d(x_p, x_{p+k}) < \varepsilon$, which means that $\{x_n\}$ is a Cauchy sequence.∎