

Another example

Solve for a root of $x^3 + x^2 - 1 = 0$ between 0 and 1. Note that if $f(x) = x^3 + x^2 - 1$, then $f(0) = -1 < 0 < 1 = f(1)$.

Trial and error yields the equivalent expression

$$f(x) = 0 \Leftrightarrow x = \sqrt{\frac{1}{x+1}} = g(x)$$

Now $f(0.5) = -0.625$, so the root actually lies between $\frac{1}{2}$ and 1. Check that $|g'(x)| \leq \sqrt{\frac{2}{27}}$ on this interval. If we set $x_0 =$

and $x_{n+1} = g(x_n)$, then we get (to 7 places)

$$x_2 = 0.7653669$$

$$x_4 = 0.7553612$$

$$x_6 = 0.7549000$$

$$x_8 = 0.7548787$$

$$x_{10} = 0.7548777$$

(and further approximations to 7 places are the same as x_{10}).