Math 145B
Spring 2017

UPDATED GENERAL INFORMATION — APRIL 10, 2017

Second countable spaces

Topological spaces with countable bases of open subsets (= *second countable spaces*) are an important class of examples. In particular, every subset of $\mathbb{R}^n$ is second countable. More complete information on such spaces is given in Section VI.1 of `gentop-notes.pdf`, which is in the course directory. However, we should note that the following passage on page 91 is potentially misleading:

> We have already noted that continuous functions from the unit interval to a Hausdorff space are completely determined by their restrictions to the rational points of the interval. In fact, this property holds for all subsets of Euclidean spaces.

As written, the second sentence might be interpreted to read that if $A \subset \mathbb{R}^n$, then a continuous function from A into a Hausdorff space Y is completely determined by its restriction to $A \cap \mathbb{Q}^n$, but this is easily seen to be false for some choices of A. For example, this is clearly the case if $A \cap \mathbb{Q}^n$ is empty, as it is when A is the set of points which have at least one irrational coordinate. There are two ways to fix this. If A is a subset which contains a nonempty open dense subset U, then $U \cap \mathbb{Q}^n$ is dense in A, so in this case continuous functions into Y are indeed determined by their restrictions to $U \cap \mathbb{Q}^n$. More generally, if $A \subset \mathbb{R}^n$ is arbitrary, then there is a countable subset $D \subset A$ such that continuous functions into Y are determined by their restrictions to D.

Exercises on connected and arc components

Here are some further questions to consider:

> Describe the connected components for the set $(\mathbb{R} - \{\mathbf{0}\}) \times (\mathbb{R} - \{\mathbf{0}\})$ (with the subspace topology it inherits from $\mathbb{R}^2$).
>
> If $f; X \to Y$ is continuous and C is a connected/arc component of X, explain why $f[C]$ is contained in a connected/arc component of Y.
>
> Prove or give a counterexample: If C is a connected component of X, then C is an open and closed subset of X.
>
> Suppose that X is a union of n connected/arcwise connected subsets $A_1, \cdots A_n$. Explain why X has at most n connected components/arc components. [This has two parts, one for connected spaces and connected components, and the other for arcwise connected spaces and arc components.]

The files `polya.pdf`, `math205Asolutions00.pdf` and `mathproofs.pdf` contain remarks which might be helpful in attempting to solve these and other exercises.