Math 145B
Spring 2017

UPDATED GENERAL INFORMATION — JUNE 2, 2017

The final examination

This will take place in Sproul 2361 on **Thursday, June 15,** from 11:30 AM to 2:30 PM. It will consist of six questions and is designed to take between an hour and a half to two hours; however, you can take the entire three hours if you choose to do so.

Some problems will be taken from the exercises in the course directories, both for the individual units and the files in the subfolder `review+exams.pdf`. There will probably also be at least one problem involving the fundamental concepts of homotopy as discussed in Sectiions III.1 and III.2 of the online notes. Here are some additional problems, some of which will appear on the exam, possibly in a slightly different or simplified form.

1. Let X be a discrete topological space. Prove that if X is given the usual metric with $d(u,v) = 1$ if $u \neq v$, then X is complete with respect to this metric, and give an example of a metric on X such that X is not complete. [*Hints:* For the first part, what happens if we have a Cauchy sequence $\{a_n\}$ and $d(a_n, a_m) < \frac{1}{2}$ for $n, m \geq M$? For the second part, you can find an example where X is a subset of the unit interval.]

2. Suppose that $X \subset \mathbb{R}^3$ is a connected graph. Prove that X is homeomorphic to a quotient space of the closed unit interval. [*Hint:* Look at the file `aabUpdate08.145B.s17.pdf` for some standard examples.]

3. Suppose that $X \subset \mathbb{R}^3$ is a connected graph, and suppose we are given an edge path $E_1 ... E_r$ in X. Prove that $P = \cup_j E_j$ is connected.

4. Let (X, x_0) be a simply connected pointed space (arcwise connected + trivial fundamental group). Prove that (X, x_0) and $\big(X \times S^1, (x_0, 1)\big)$ are not base point preservingly homotopy equivalent.

5. Suppose that $(X_n, \mathcal{E}_n)$ is the complete graph on n vertices, where $n = 3$ or 4. Determine, which, if any, of these graphs has an Euler path; in other words, an edge path which contains each edge exactly once. Formulate and prove a generalization to arbitrary values of n.

6. Let $(X, \mathcal{E})$ be a connected graph such that every vertex lies on exactly k edges, where $k \geq 1$. Choose a linear ordering ω of the vertices, and let $\mathbb{F}$ be a field. Prove that k and the number V of vertices cannot both be odd, and derive a formula for the dimension of $H_1(X, \mathcal{E}, \omega; \mathbb{F})$ which depends only on V. [*Hint:* Look at the propositions at the end of Section 5.3 in the notes.]