NAME: ______________________________

Mathematics 145B, Spring 2015, Examination 1

Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.

#	SCORE
1	
2	
3	
4	
TOTAL	

1. [*25 points*] (a) If X is a topological space and C is a connected component of X, explain why C is a closed subset of X.

(b) Give an example of a topological space X and a connected component C such that C is not an open subset of X.

(c) Give an example of a topological space X and an arc component A such that A is not a closed subset of X.

2. [*25 points*] Let X be a complete metric space. If $\{x_n\}$ is a sequence of points in X such that for each n we have $d(x_{n+1}, x_{n+2}) \leq \frac{1}{2}\, d(x_n, x_{n+1})$, then an induction argument, the Triangle Inequality and the formula for summing a geometric progression imply that

$$d(x_n, x_{n+k}) \quad \leq \quad \frac{1}{2^{n-1}}\, d(x_0, x_1)$$

for all nonnegative integers n and k. Using this, explain why $\{x_n\}$ has a limit in X.

3. [*30 points*] For each of the following, determine whether the statement is true or false, and give brief explanations for your answer (one or two sentences should be enough).

(a) If X is a compact topological space and $\mathcal{R}$ is an equivalence relation, then the set of equivalence classes $X/\mathcal{R}$ is compact if it is given the quotient topology.

(b) If X is a Hausdorff topological space and $\mathcal{R}$ is an equivalence relation, then the set of equivalence classes $X/\mathcal{R}$ is Hausdorff if it is given the quotient topology.

(c) There is an equivalence relation $\mathcal{R}$ on the unit interval $[0,1]$ such that $[0,1]/\mathcal{R}$ with the quotient topology is homeomorphic to the circle S^1.

4. [*20 points*] Let X be a topological space, let Y be a convex subset of $\mathbb{R}^n$ for some n, and let $f, g : X \to Y$ be continuous mappings. Prove that f and g are homotopic.

Additional sheet for use if needed.