NAME: ______________________________

Mathematics 145B, Spring 2015, Examination 2

Work the first four questions for full credit, and unless indicated otherwise give reasons for your answers. The fifth question is an extra credit problem. Point values for individual problems are indicated in brackets.

#	SCORE
1	
2	
3	
4	
5	
TOTAL	

1. [*30 points*] (a) Let $f : (S^1, 1) \to (S^1, 1)$ be the mapping $f(z) = z^2$ where squaring refers to complex multiplication on $\mathbb{C} = \mathbb{R}^2$. Using fundamental groups (or some other valid method), prove that f is not base point preservingly homotopic to the identity map.

(b) Prove that S^1 is not a retract of D^2.

2. [*20 points*] Let $n \geq 2$. Prove that S^{n-1} is a strong deformation retract of $\mathbb{R}^n - \{\mathbf{0}\}$.

3. [*20 points*] Determine whether each of the following statements is true or false, and give reasons for your answer.

(a) If (X, x) is a pointed space and $f : X \to Y$ is a continuous mapping of topological spaces, then there is some $y \in Y$ such that f defines a base point preserving continuous mapping from (X, x) to (Y, y).

(b) If (Y, y) is a pointed space and $f : X \to Y$ is a continuous mapping of topological spaces which defines base point preserving continuous mappings from (X, x_0) and (X, x_1) to (Y, y) for $x_0, x_1 \in X$, then $x_0 = x_1$.

4. [*25 points*] Let $(X, \mathcal{E})$ be a complete graph for the vertices a, b, c, d; in other words, for each pair of vertices there is a unique edge containing both of them. Explain why X is connected and compute $H_1(X, \mathcal{E}^\omega; \mathbb{F})$ where ω is the alphabetical ordering of the vertices and $\mathbb{F}$ is a field (the answer does not depend upon which field is chosen).

5. [*15 points*] For each of the graphs on the next page, determine whether an Euler path exists and give reasons for your answer. If an Euler path exists, there is no need to construct one explicitly for full credit.

replace this page with a drawing

Additional sheet for use if needed.