Alternate solution to Problems $5(c)$ and $5(d)$ in Exam 3

The positive examples in `exam3s17key.pdf` may seem too trivial or artificial, and they do not use the hint included in the exam, so here is an alternate approach which has a less trivial example and does involve the hint.

Example A. Let $\mathcal{R}$ be the equivalence relation on $\mathbf{I}$ whose equivalence classes are the two point set $\{0,1\}$ and the one point sets $\{t\}$ where $0 < t < 1$, so that $\mathbf{I}/\mathcal{R}$ is homeomorphic to S^1. Then we have the following:

A1. *This quotient space is metrizable.*

A2. *This quotient space Y has a quotient of its own $W = Y/\mathcal{E}$ which is homeomorphic to $\mathbf{I}$.*

Statement A1 follows because the quotient space $\mathbf{I}/\mathcal{R}$ is homeomorphic to S^1. To prove Statement A2 using the hint, first note that S^1 is homeomorphic to the circle C with radius $\frac{1}{2}$ and center $\left(\frac{1}{2}, 0\right)$, so it suffices to show that there is an equivalence relation $\mathcal{E}'$ on C such that the quotient $C/\mathcal{E}'$ is homeomorphic to $\mathbf{I}$. This in turn reduces to finding a continuous onto mapping from C to $\mathbf{I}$. One easy way of constructing such a mapping is to take the function f which sends (x, y) to x. It is a straightforward exercise to check that f is continuous and its image is equal to $\mathbf{I}$.