Corrected hint for a problem in `aabUpdate12.145B.s17.pdf`

The hint for Problem 2 in the file `final2017-prep1.pdf` does not suffice, but it can be modified to do so.

The problem with the hint. In order to prove that "the [original] graph $[X]$ is homeomorphic to a quotient of a star shaped graph" whose vertices are $\{v_0, \cdots, v_m\}$ and edges are given by v_0v_j where $1 \leq j \leq m$, we need to know that every edge of the graph lies on one of the subgraphs $|P_j|$ determined by reduced edge paths P_j joining v_0 to v_j (where $1 \leq j \leq m$) that were chosen at the first step. **However, this is not necessarily true.** All we can say is that the subgraph $\cup_j |P_j|$ is homeomorphic to a quotient (because there is a continuous onto map from the star shaped graph to the subgraph).

The simplest example with $\cup_j |P_j| \neq X$ is given by the edges of a triangle with vertices v_0, v_1, v_2: If we can take P_j to be v_0v_j for $j = 1$ or 2, then the edge v_1v_2 does not lie in the subgraph $|P_1| \cup |P_2|$.

One way to fix this. Instead of choosing just one reduced edge path joining v_0 to v_j for each $j > 0$, take **all** reduced edge paths joining v_0 to the other vertices, and consider the star shaped graph which consists of edges which all contain a given vertex x_0, with one edge for each reduced edge path as above. Then it is fairly straightforward to construct a continuous mapping from the new star shaped graph to the original graph X.

The crucial step now is to prove that this continuous mapping is onto. If we can do that, then the final step in the hint from `final2017-prep1.pdf` — showing that the star shaped graph is a quotient of a closed interval using the hint in `aabUpdate12.145B.s17.pdf` — will complete the proof that X is homeomorphic to a quotient of the closed unit interval.

In order to show the mapping is onto, it is enough to show that every edge of the graph lies on some subgraph determined by a reduced edge path starting at v_0. This can be done as follows: Suppose that K is an edge whose vertices are y and z; if v_0 is a vertex of K then by construction K is a reduced edge path and hence it lies in the image of the mapping from the star shaped graph, so let's assume that v_0 is not a vertex of K. Now let $P = E_1 ... E_r$ be a reduced edge path from v_0 to y, and consider the edge path P_1 given by the concatenation $P + K$. If z is not a vertex of some E_j then this concatenation is a reduced edge path from v_0 to z, and therefore K lies in the image of the map from the star shaped graph to X. On the other hand, if z is the vertex v_j for some E_j, we can truncate P at v_j to obtain a new reduced path P', and the concatenation $P' + K$ will then be a reduced edge path joining v_0 to y. This completes the proof of the statement in the first sentence of this paragraph, and hence completes the proof of the exercise.

Important comment. The preceding argument seems like too much to ask for on an exam for this course, so here is an **alternative problem** which is **strongly recommended**:

7. Let $(X, \mathcal{E})$ be a graph, and let $P = E_1 \cdots E_r$ be an edge path (not necessarily reduced!) in X. Prove that the subgraph $|P|$ defined by P is homeomorphic to a quotient space of the closed unit interval. [*Hint:* Recursively define a continuous mapping which sends $[0, \frac{k}{r}]$ to $E_1 \cdots E_r$ for $1 \leq k \leq n$.]