SOLUTIONS TO EXERCISES FOR

MATHEMATICS 205A

Fall 2014

General Remarks

The main objective of the course is to present basic graduate level material, but an important secondary objective of many point set topology courses is to is to build the students' skills in writing proofs and communicating them to others. Higher skill levels are needed because proofs in graduate level mathematics courses are frequently longer, more abstract and less straightforward than their counterparts in undergraduate level courses.

The files with solutions to exercises are named `solutions*.pdf`, where $*$ is some number, perhaps followed by a letter to indicate supplementary content like drawings to accompany arguments. Exercises marked with one or two asterisks should be viewed as having lower priorities unless their solutions are specifically assigned as readings for the course.

These solutions are posted mainly for students to compare with their own efforts and to determine whether their solutions are correct or can be improved upon (compensating in part for the lack of homework grading personnel), but in some cases the solutions might also be useful if and when a student reaches an impasse. At some points, solutions to some of the more complicated exercises may be parts of reading assignments. However, as a general rule **the solutions should not be viewed as an excuse for not trying to work the exercises, especially those that are specifically assigned in course announcements** (*i.e.*, the `aabUpdate`$*$ files in the course directory). Problems at the level of those in the assigned exercises will appear on both course and qualifying examinations, so it is important for students to be able to work out solutions on their own.

Suggestions for working exercises

The directory file `polya.pdf` contains a systematic and general list of suggestions for approaching and solving mathematical problems, and the file `mathproofs.pdf` — which was written for undergraduate level courses but is still relevant at higher levels — discusses the more formal aspects of mathematical proofs. Here are a few additional comments:

1. Many exercises can be solved by imitating arguments in the course text or notes, so it is usually worthwhile to see if an exercise can be analyzed by modifying a previously seen argument. The following quotation from the *Poetics* of Aristotle (384–322 B.C.E.; see Section I, Part IV) seems appropriate:

> *The instinct of imitation is implanted in man from childhood, one difference between him and other animals being that he is the most imitative of living creatures, and through imitation learns his earliest lessons.*

The differences between human and animal behavior in some species might not be as vast as they seemed to the ancient Greeks, but the passage still reflects the importance of imitation in human thought and action.

2. The first efforts (and in many cases subsequent efforts!) at solving exercises will not necessarily be as clear or polished as some proofs and solutions in textbooks or the files posted to the course directory. Often the first attempts to find solutions are at least somewhat messy, and they usually get better as a result of increased experience, skills, and trial and error. The following quotation due to A. S. Besicovitch (1891–1970) summarizes everything in a somewhat ironic manner:

> *A mathematician's reputation rests on the number of bad proofs he has given.*

3. It is usually good to try anything that might work rather than not getting started with work on a problem. This advice reflects a frequently repeated quotation due to L. R. ("Yogi") Berra (1925–):

> *When you come to a fork in the road ... Take it.*

Likewise, if a solution to a problem is not apparent after some thought, it is often worthwhile to be systematic and look at *everything* that can be said about the given situation, no matter how insignificant it might seem at first. This is summarized in a quotation from one of the *Sherlock Holmes* stories by A. C. Doyle (1859–1930):

> *You know my method. It is founded upon the observation of trifles.*

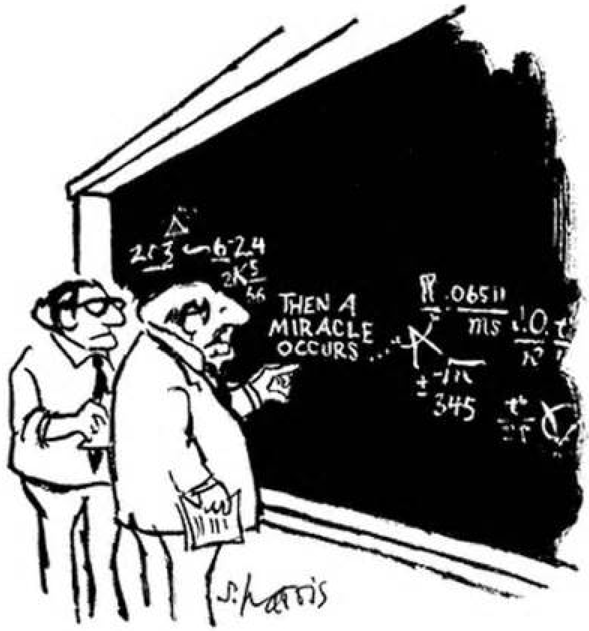
4. Although rigorous mathematical proofs must be expressed in words and symbols rather than pictures, in many cases good (or even not so good) drawings are extremely helpful — sometimes even indispensable — sources for insights which can suggest an approach to proving a mathematical statement. **This is particularly true for courses with substantial amounts of geometric content like Mathematics 205A.** The following two quotations reflect this point:

> *Geometry is the science of correct reasoning [based] on incorrect figures.* — G. Polyá (1887–1985)

> *For me, following a geometrical argument purely logically, without a picture for it constantly in front of me, is impossible.* — F. Klein (1849–1925)

5. The first step in putting together a logical argument is to come up with something that appears to be correct, but one important test of an argument's clarity and validity is to *refine the ideas so that they become equally clear and convincing to someone else who is reasonably well-informed about the subject.* This generally requires healthy doses of skepticism and self-criticism; these can certainly be overdone, but initially most of us need to be more concerned about not going far enough.

The next page contains two well-known satirical cartoons by Sidney Harris (1933–) on convincing arguments. We should also note that that there are several anthologies of Harris' excellent cartoons on humorous aspects of scientists and their work, and they are definitely worth reading and viewing.



"I think you should be more explicit here in step two."