Math 145B
Spring 2017

UPDATED GENERAL INFORMATION — APRIL 20, 2017

The first quiz

This will cover material through Section I.2. You should know the definitions of basic concepts like sequential compactness, limit point compactness, second countability, the Lindelöf Property, Cauchy sequences, and completeness. Also, you should know the statements of basic results like the equivalence of compactness and sequential compactness for metric spaces, the fact that a subset A of a complete metric space X is complete in its own right if and only if A is closed in X, and the fact that a compact metric space is complete. Here is a theorem which involves many of these concepts; you should at least be able to describe the steps in the proof:

Proposition. *Let X be a metric space in which every closed and bounded subset is compact. Then X is complete.*

Here are some hints: Let $\{a_n\}$ be a Cauchy sequence in X, and let A be the closure of the set $\{a-1, a_2, \cdots\}$. Use the boundedness of the Cauchy sequence to derive the boundedness, and hence also the compactness, of A.

A variation of this result is to show that X is complete if every bounded infinite sequence in X has a convergent sequence (which we already know is true if X is the real line).