Math 145B
Spring 2017

UPDATED GENERAL INFORMATION — MAY 22, 2017

Letters as quotients of the unit interval

One very tangible class of quotient spaces is given by letters of the (Latin) alphabet, the usual Hindu-Arabic numerals, and other familiar symbols. Here are a few particularly significant examples:

A B D E H K P Q R X Y 4 8

In more accessible terms, the problem amounts to drawing letters and other figures without lifing the pencil or pen from the paper, where it is permissible to retrace something already drawn. For example, if we consider the letter T, then we can do this by starting at the base, moving up to the top, then going to the left to draw the left hand part of the letter's top, and finally going to the right, retracing the left hand part of the top and then continuing to complete the right hand part of the top. We can view this drawing as a continuous curve from the unit interval $[0,1]$ onto T, and this curve is a quotient map (it's closed by compactness and the fact that the figure T may be viewed as a subset of the plane). The equivalence classes are sets of points which have the same image in the figure T. If we suppose that the curve is parametrized linearly on pieces, with $\left[0,\frac{1}{4}\right]$ going from the bottom of the top of the middle stem, $\left[\frac{1}{4},\frac{1}{2}\right]$ going to the left at the top, and $\left[\frac{1}{2},1\right]$ going all the way across the top, then the equivalence classes containing more than one point are exactly the following:

All pairs $\{t, 1-t\}$, where $\frac{1}{4} \;\le\; t \;<\; \frac{1}{2}$

Clearly there is one problem of each type for each letter of the alphabet and each numeral, and if this does not provide enough examples one can take connected examples from other alphabets such as Greek or Cyrillic (excluding disconnected examples like the Greek Ξ). More generally, one can do the same thing for the connected graphs which will be discussed in Unit V of this course.

The preceding question leads directly to the problem of characterizing those topological spaces which can be realized as topological quotients of the closed unit interval $[0,1]$; this question is discussed in `intro2topA-13.pdf`.