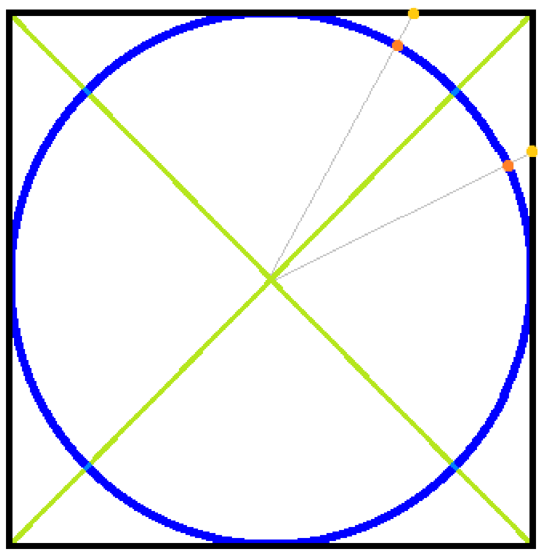
Drawing for Exercise IV.4.5(c)

The illustration shows the standard method of proving that a circle is homeomorphic to the boundary of a sphere by means of ***radial projection.*** We start with a circle and circumscribe a square about it such that the sides of the square are parallel to the coordinate axes. Then we consider the linear ray starting at the center and going through a point of the circle. This ray will meet the boundary of the square at some point as one travels outward on the ray from the point on the circle, and the point at which the ray meets the boundary of the square is called the ***radial projection of the original point onto the boundary of the square.***



In the drawing, a point on the circle is a dark orange dot and its image is a light orange dot. An inverse mapping is apparent from the picture; we merely reverse the preceding construction, starting with a point on the boundary of the square and taking the point on the line segment joining that point and the center at which the segment meets the circle. To show that we really obtain a homeomorphism in this manner, as usual it is necessary to translate the picture into algebraic formulas.

This result extends to higher dimensions. The unit sphere in $n-$ space is homeomorphic to the boundary of the hypercube, which consists of those points whose coordinates are all between -1 and $+1$. However, one needs a different proof. Details can be found on pages $84-85$ of the following document:

http://math.ucr.edu/~res/math205B-2012/algtop-notes.pdf