Addendum to I.1

In the proof of Corollary I.1.5, the following observation is very helpful:

LEMMA I.1.6. *If X is a metric space and $A \subset X$ such that $p \in X$ is a limit point of A. Then for every $\varepsilon > 0$ the set $N_\varepsilon(p) - \{p\}$ contains infinitely many points of A.*

The result also holds more generally for spaces in which every one point subset is closed (hence in all Hausdorff spaces), but the case of metric spaces is all we shall need.

Proof. Suppose that $(N_\varepsilon(p) - \{p\}) \cap A$ contains only finitely many points, and write them as a finite sequence $a_1, \cdots a_m$. Let h be half the minimum of the distances $d(p, a_i)$; this number is positive because there are only finitely many distances and they are all positive (because $p \neq a_i$ for all i). Then we have $(N_h(p) - \{p\}) \cap A = \emptyset$, which contradicts the assumption that p is a limit point of A.■