

Note In order to complete the justification of the square root algorithm, we should check that $T(a) \leq a$. Since T is increasing for $x \geq a$, this will imply that T maps $[\sqrt{a}, a]$ into itself. Recall that $a > 1$.

By definition, $T(a) = \frac{1}{2}(a+1)$, so we have $T(a) = \frac{1}{2}a + \frac{1}{2} < \frac{1}{2}a + \frac{1}{2}a < a$,
($a > 1$)

which is all we need to prove. ■