

**ADDITIONAL SOLUTION TO EXERCISES FOR
MATHEMATICS 145B — Part 3a**

Spring 2017

III.2 : Homotopy equivalence

Problem from Munkres, § 58, pp. 366 – 367

6. Since A is a retract of X , there is a retract mapping $f : A \rightarrow X$ and a one-sided inverse $g : X \rightarrow A$ such that $g \circ f = \text{id}_A$. Since X is contractible there is a point $p_0 \in X$ and a homotopy $H : X \times [0, 1] \rightarrow X$ such that $H(x, 0) = x$ and $H(x, 1) = p_0$ for all $x \in X$. Define a homotopy

$$K : A \times [0, 1] \longrightarrow A$$

by $K(a, t) = g \circ H(f(a), t)$. The definitions then imply that $K(a, 0) = g(f(a)) = a$ and $K(a, 1) = g(p_0)$. Therefore A is also contractible. ■