# Illustrations for Section VII.4

These drawings are cited in the proofs that the utilities network and the complete graph on five vertices cannot be topologically embedded in the plane (or the 2 – sphere).

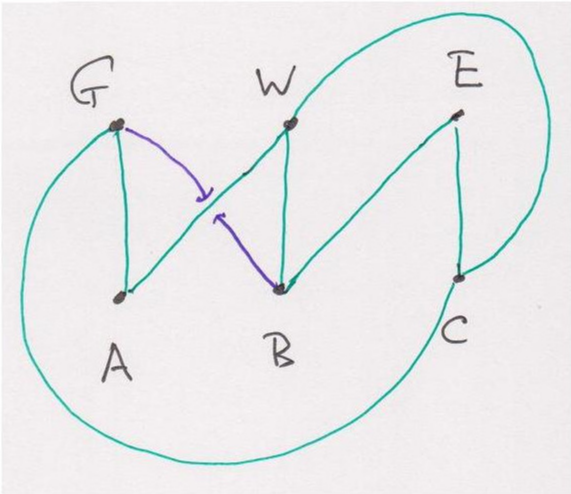


**Figure 1**

This is a drawing of the utilities network. Notice that the edge joining **A** to **W** passes over the edge joining **B** to **G.** One goal of Section VII.4 is to prove mathematically that one cannot move the connecting lines between the vertices so that no edge passes over or under another.

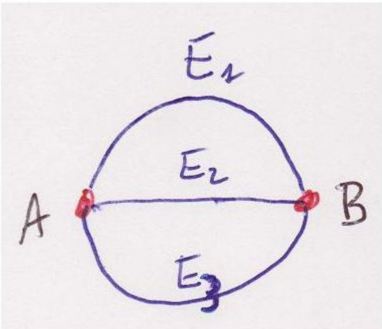


**Figure 2**

This is a standard topological embedding of a theta space in the plane. Observe that its complement has three components such that the boundary of each component is a union of two edges in the theta space, and different components have different boundaries.

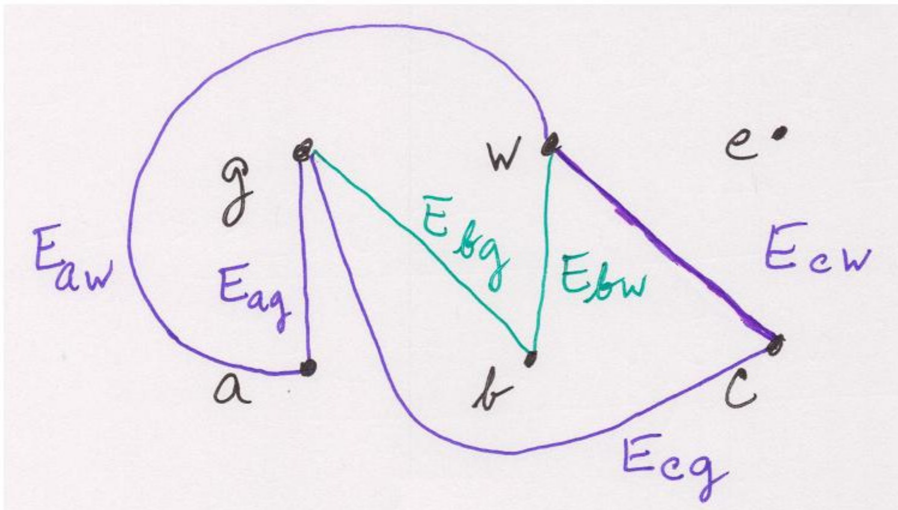


**Figure 3**

This is the subgraph **S** of the utilities network obtained by removing the three edges which have ***e*** as an endpoint (the vertex ***e*** is also removed from the graph but the other endpoints of the removed edges are in the subgraph)**.** Observe that this graph is homeomorphic to a theta space**.** Two of the three edges of the theta space are colored in purple, and the third is colored in green**;** the union of the two purple edges is a simple closed curve **K.**

In Figure 3 the vertices ***e*** and ***b*** lie in different components of the complement of **K,** and if this is the case then the picture suggests that a curve joining them must pass through **K.** It is possible that ***e*** lies in one of the other two components of the complement of **S,** but if ***e*** lies in the component bounded by the closed curve in **S** passing through ***b*** and ***c*** then the picture suggests that a curve joining ***e*** to a must pass through **S,** while if ***e*** lies in the component bounded by the closed curve in **S** passing through ***b*** and ***a*** then the picture suggests that a curve joining ***e*** to ***a*** must pass through **S.** This is not quite the argument in the formal proof given in the notes**,** but one can use the results in Section VII.4 in the notes to write a rigorous proof along these lines**.**

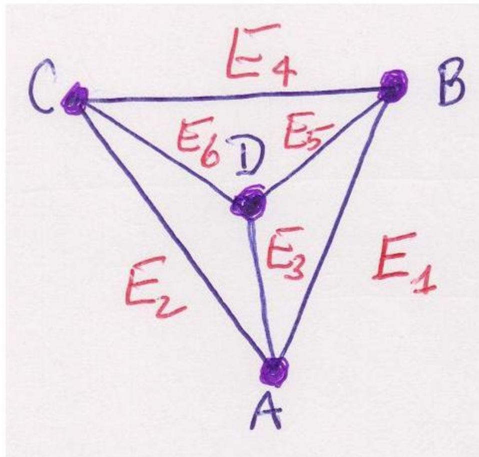


**Figure 5**

This is the standard linear embedding of the complete graph on four vertices in the plane
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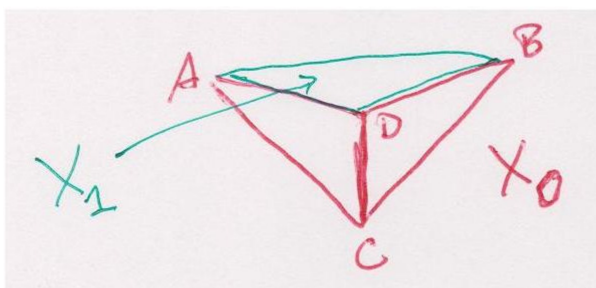


**Figure 6**

The complete graph on four vertices can be decomposed as a union of two subgraphs, one of which is a simple closed curve and the other of which is a theta space; the intersection of these subgraphs is homeomorphic to a closed interval.